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Risky higher education and subsidies

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Abstract

Tertiary education in the U.S. requires large investments that are risky, lumpy, and well-timed. Tertiary education is also heavily subsidized. Our model suggests that despite adverse selection arising from encouraging poorly prepared students to enroll, observed collegiate subsidies improve outcomes substantially relative to the fully decentralized case. This result occurs because increases in subsidy rates for college education generate reductions in college failure risk without altering mean returns. We find that this mechanism is robust, and that tertiary subsidy rates well in excess of those observed in the U.S. can be justified by failure risk alone.

JEL classification: E60; I22

Keywords: Human capital risk; Heterogeneous agents; College education

1. Introduction

Tertiary education in the U.S. requires large investments that are risky, lumpy, and well-timed. Tertiary education in both the U.S. and the rest of the OECD is also heavily subsidized, typically in the range of 30–100%. The outcomes generated by

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this large and uncertain investment are income streams that do not depend on initial wealth, but rather are of given absolute size. For standard preferences, this implies that poor households will be less willing to attempt college, all else equal, than their wealthier counterparts. In turn, for a given skill premium, such households will remain poorer over generations as subsequent cohorts become increasingly unwilling to take the risk. Therefore, cross-sectional inequality will be larger, and intergenerational incomes more persistent, than they would otherwise be. This mechanism raises several natural questions that, to our knowledge, are unanswered. Namely, how does higher education subsidy policy affect welfare and prices such as the skill premium? How do subsidies alter output and consumption levels? Finally, are subsidies important in altering long-run inequality and the dynamics of the intergenerational earnings process? The contribution of this paper is to provide the first quantitative assessment of how subsidies affect the intergenerational earnings process and consequently long run income, wealth and consumption inequality.

Subsidies make collegiate investment more attractive, especially to low wealth households, by reducing both the direct cost of college and the cost of making a risky investment in human capital. When investing in education is risky and uninsurable, even small initial changes in college investment may generate large changes in allocations. A primary force by which such a process is set in motion is through a mechanism noted in Ljungqvist (1995). Specifically, a high skill premium implies that collegiate outcomes can result in large changes in consumption and wealth accumulation. If collegiate investment risk were insurable, households would respond to increases in skill premia with increases in enrollment. However, when college investment is risky, an increase in the skill premium also implies an increase in college investment risk, all else equal. Therefore, relative to full insurance, the 'pecuniary' effect introduced jointly by uninsurable risks and the skill premium reduces the enrollment incentives created by a given skill premium. Conversely, any increase in enrollment will lower skilled wages and increase unskilled wages, all else equal. To the extent that such a compression of the skill premium is even approximately mean preserving, the riskiness of collegiate investment falls. This reduction in risk in turn produces further incentives to enroll, making subsidies a potent policy instrument in starting a 'virtuous cycle' of educational attainment.

There is a second important force by which subsidies affect outcomes. This arises from strong evidence that college-educated parents produce or raise children who are substantially better prepared for college. For example, Ishitani and DesJardins (2002); Warburton et al. (2001); Chen (2004) and Horn and Nunez (2000) all document the presence of strong 'first-generation' (students whose parents did not complete college) effects, even after controlling for family income. Therefore, the effects of subsidies are likely to extend beyond the lifetime of an individual, influencing long run income and wealth inequality.

Third, it is known at least since Aiyagari (1994) that stationary equilibria in environments with uninsurable risks display overinvestment in physical capital in the sense that the interest rate will be strictly below the rate of time preference. Education subsidies lower this excess capital via two different channels. First, subsidies decrease the private cost of education investment. Therefore, households do not have to save large amounts against the possibility of having to send an academically able child to college. Second, because lower costs leave households better able to risk college investment, the precautionary demand for assets falls. A lower capital stock raises the rate of return on savings and improves consumption smoothing.

While subsidies have the potential to generate large benefits, they introduce two potentially large types of costs. First, subsidies may generate non-trivial adverse selection by encouraging progressively more poorly prepared students to attend. Second, subsidies must be financed via taxation and when markets are incomplete, even lump-sum taxes are distortionary.

Our model suggests that observed collegiate subsidies improve welfare substantially relative to the fully decentralized (zero subsidy) outcome. The primary force driving our welfare conclusions is that increased subsidies turn out to allow *nearly mean preserving reductions in college risk*. Subsidies increase unskilled wages and lower skilled wages roughly proportionally as more people attend college, narrowing the skill premium and lowering the consequences of collegiate failure. Therefore, the mean return to education, given average failure risk, does not change greatly across equilibria with different subsidy rates. Subsidies therefore help smooth consumption, lower skill premia, increase interest rates as precautionary savings fall, lower the inequality of both consumption and wealth, and increase intergenerational income mobility.

We also find in the benchmark case that even though physical capital shrinks with subsidies, mean consumption does not fall, but rather rises slightly. Greater consumption is feasible because the increase in human capital is efficiency enhancing, as it raises steady-state output with less physical capital. Our model thus quantifies the extent of underinvestment in risky human capital emerging from current and proposed subsidy rates. In our benchmark case, risky human capital accumulation falls well short of the welfare maximizing value. The risk of failure in college is by itself sufficient to explain subsidies to college education observed in the U.S. and other OECD countries. The largest welfare gains from subsidies occur at rates below 40%, with much smaller gains from further subsidization. This feature provides insight into why OECD countries simultaneously display both uniformly high subsidies as well as the absence of explicit private insurance against failure: additional insurance, private or public, is not very useful. An important feature of our study is that we focus on the role of subsidies in mitigating risk arising in the *acquisition* of human capital. We therefore shut down other sources of benefits from subsidies, such as externalities and endogenous growth. We also do not impose credit constraints on student loans, as the U.S. government provides full loan guarantees on college loans via such programs as PLUS. Interestingly, despite removing most obstacles to financing college, the government continues to subsidize it heavily.

In the U.S. and other OECD nations, college education is a risky investment. Using NLSY data, Altonji (1993) finds that of high school students intending to *complete* college in 1972, 90% did enroll, yet only 58.1% had completed 7 years later. The risk of failure is also present for the seemingly well-prepared. For example, at colleges classified as 'Highly Selective' by the ACT, the dropout rate averages roughly 19%. Most striking is that even for those who had taken more than one advanced placement test, Berkner et al. (2003), Table 10.3.A., finds that the drop-out rate does not fall to zero, but only to about 20%. In recent work, Chen (2004) finds that roughly 16% of 1997 high-school graduates with test scores in the top quartile did not even attempt college.[1] Estimates of Willis (1986) and Card (1995), for example, show the rate of return on college to be between 8% and 13%, a significantly higher rate than on financial assets. Given the risks, it appears that a portion of this high rate of return is a risk premium, and Chen (2004) estimates the risk premium associated with a 4-year college education to be one-fourth of the total return.

College is also a lumpy investment. As the classic study of Levhari and Weiss (1974) notes: "In fact, the length of schooling itself may be random, since the ability to complete a given schooling program is also to some extent unknown." However, they do not model this source of uncertainty in their analysis, while this is a key aspect of our study. Most students who fail to complete college do so only after spending a substantial amount of time enrolled. Berkner et al. (2003), Table 5.0B, shows that while the dropout rate exceeds 10% in the first year, it is even higher at the end of the second year, and is still nontrivial, at over 5%, in the third and fourth years of college. Stinebrickner and Stinebrickner (2003) use a different data set and find that the median time to dropping out is 2 years. Worse yet is that dropping out, even after several years in college, is typically very damaging. Hungerford and Solon (1987) and Card and Kreuger (1992) both find that returns are highest in the later years of college, and substantially greater than those obtained in the first 2 years. The risky and lumpy nature of investment in college may therefore be dissuading even talented high-school completers from attempting college at all, implying systematic underinvestment in human capital. These individual-level risks are reflected in aggregate tertiary education failure rates in the OECD that exceed 30% (Fig. 1). Berkner et al. (2003), Table 4.3A, shows that for the U.S., the unconditional dropout rate was approximately 37% for the period 1993–1998.

Given the lumpiness in collegiate investment, failure is very costly. With respect to size, in the U.S., for example, the median resource cost of college in tuition and fees, room, and board exceeds $60,000.[2] Even beyond this large explicit cost of college, the opportunity cost of lost wages is large, at approximately $17,000 per full-time student annually in the U.S.[3] Also, the current wage premium for college education of 1.6 implies very large differences in mean lifetime income between those who complete college and those who do not.

The opportunity cost of higher education rises sharply with age, as households form and children are produced. Therefore, there is typically only a narrow window of time available for college education. For example, Cameron and Heckman (2001) show that approximately 90% of college entry occurs, if at all, within 2 years of

[1]Work of Cameron and Heckman (1999), Carneiro and Heckman (2002), and Keane and Wolpin (2001) suggests that short-term credit constraints are not important enough to explain this fact.

[2]NCES (1995), Condition of Education (COE).

[3]Source: http://www.census.gov.

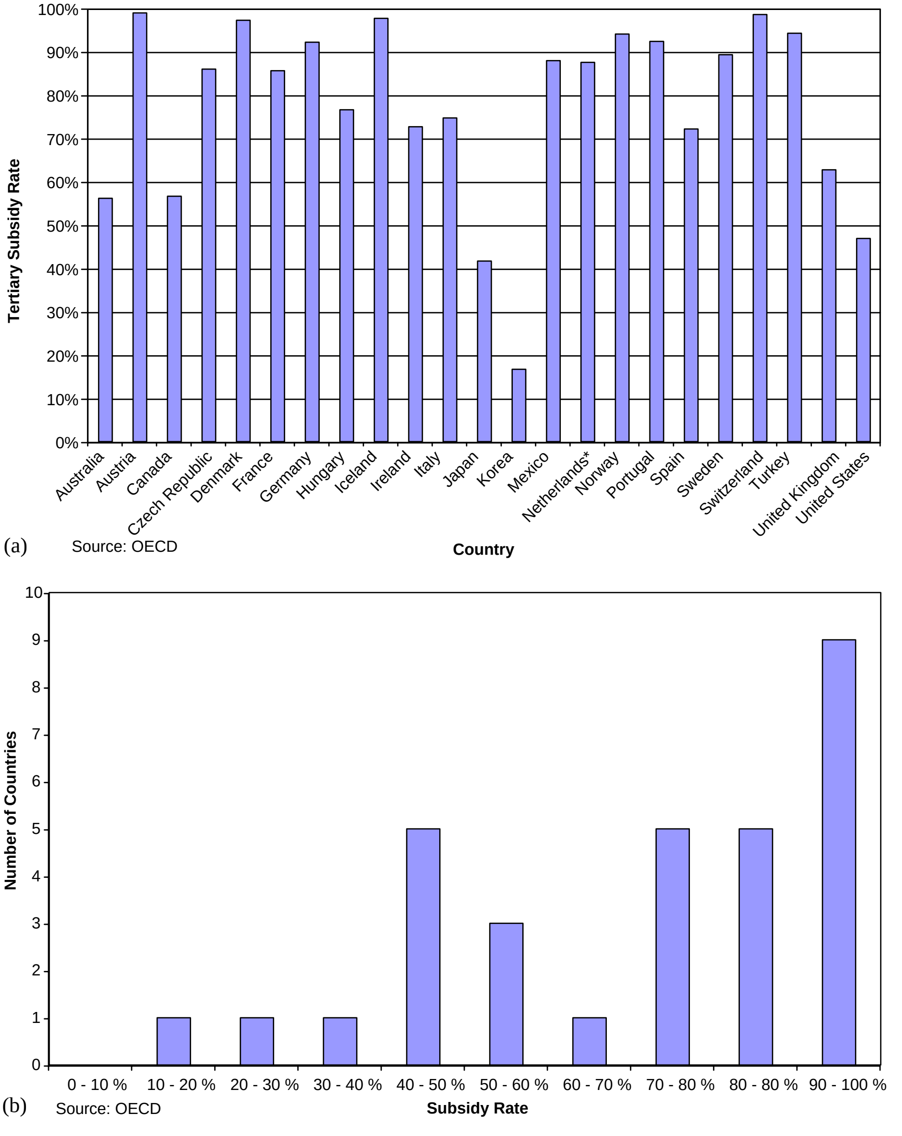


Fig. 1. OECD tertiary: (a) subsidy rates and (b) education subsidy rates.

completing high school. It is also not easy to overcome an initial failure in college via re-enrollment at a later time, nor is it costless to delay entry. Ahlburg et al. (2002) find that even after accounting for endogeneity in the decision to delay college entrance, the likelihood of dropout rises by 32% for high school graduates who postpone college entry. In such an environment, relatively poor families—even those with well-prepared children, may choose not to send them to invest in a risky college education, even if student loans are assumed to be comprehensive enough to allow all households to feasibly do so. Such outcomes exacerbate cross-sectional inequality and endogenous intergenerational persistence in income and wealth.

In addition to the risk faced by those trying to acquire a college education, U.S. households bear a considerable amount of uninsurable idiosyncratic labor income risk. The empirical tests of Cochrane (1991), Hayashi et al. (1996), and Carroll (2000) all document strong departures from complete risk sharing, both between and even within families. The research of Levhari and Weiss (1974), Eaton and Rosen (1980), and Hamilton (1987) emphasizes uncertainty in the return to human capital, and the difficulties in diversifying such risk. The long-run implications of uninsurable risk are explored in recent work by Castañeda et al. (2003); Díaz-Giménez et al. (1997), and De Nardi (2004). These authors show that the high income and wealth inequality observed in the U.S. can be plausibly accounted for by parsimonious, incomplete market models, with exogenously specified earnings processes. One contribution of this paper is that our study endogenizes the mean of the household earnings process by allowing individuals to choose skills in response to stochastic income shocks, educational risk, skill premia, and subsidies.

The environment we develop is most closely related to that of Caucutt and Kumar (2003) and Restuccia and Urrutia (2004).[4] Caucutt and Kumar (2003) study a model with heterogeneity in ability, and find that subsidies have modest effects on educational attainment and welfare. Restuccia and Urrutia (2004) study an environment that distinguishes between pre-college and college investment. Differences in collegiate investments are found to account for most of the observed inequality in earnings, while early investments are found to have the largest effects on intergenerational persistence. There are important differences between our work and both of the preceding papers. First, Caucutt and Kumar (2003) and Restuccia and Urrutia (2004) prohibit borrowing to finance education. Recent work of Cameron and Heckman (1999), Carneiro and Heckman (2002), and Keane and Wolpin (2001) finds that borrowing constraints do not appear to affect college enrollment decisions. Therefore, we always allow individuals to borrow to pay for college tuition, room, and board, even though households face (state-dependent) limits on their overall debt. Second, we follow Levhari and Weiss (1974), Eaton and Rosen (1980), Hamilton (1987), Chen (2004), and others in assuming that human capital, once acquired, produces uncertain returns. Furthermore, we limit income insurance possibilities to self-insurance via capital accumulation. Conversely, both Restuccia and Urrutia (2004) and Caucutt and Kumar (2003) assume full insurance of labor income and do not allow physical capital accumulation. These differences

[4]Krebs (2003) finds a welfare improving role for state-contingent transfers in an endogenous growth model with riskless physical and risky human capital. However, in order to focus on growth and portfolio choice, Krebs' model abstracts from lumpiness, irreversibility, and timing frictions in human capital investment, as well as heterogeneity in ability. Also, Krebs (2003) studies reductions in human capital risk, once acquired. In our model, both human capital acquisition itself and the subsequent returns are risky.

allow us to study the quantitative implications of subsidies for the wealth distribution.[5,6]

Our paper is organized as follows. In Section 2, we lay out the model. Section 3 defines equilibrium and Section 4, the welfare measure. We parameterize the model in Section 5. We discuss the results in Section 6, and conclude in Section 7. The computational algorithm is detailed in Appendix A.

2. Model

2.1. Preferences, endowments, assets

The economy consists of a continuum of infinitely lived dynasties with unit mass. Each agent/dynasty has CRRA preferences over the single consumption good c:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\mu}}{1-\mu}, \tag{1}$$

where β denotes the discount factor, and μ is the coefficient of relative risk aversion.

All agents are endowed with one unit of time, which they supply inelastically. Agents supply skilled labor if they have obtained education, unskilled labor otherwise. The private cost of college is denoted by γ. The steady-state wage rate for skilled labor is denoted by $w_{\rm s}$, and the unskilled steady-state wage is denoted $w_{\rm u}$. To reflect uncertainty in the returns to human capital, agents are subject to idiosyncratic labor productivity shocks. These shocks are denoted $\varepsilon \in \{\varepsilon_{\rm l}, \varepsilon_{\rm h}\}$, where h and l denote high and low productivity states, respectively. Shocks follow a Markov process with transition probabilities $p_{\rm hh} = \Pr(\varepsilon' = \varepsilon_{\rm h} | \varepsilon = \varepsilon_{\rm h})$, and $p_{\rm ll} = \Pr(\varepsilon' = \varepsilon_{\rm l} | \varepsilon = \varepsilon_{\rm l})$. The unconditional expected labor productivity for a given agent is normalized to unity and is denoted by $\bar{\varepsilon}$.

To capture the presence of uninsurable risk, agents are limited to trade in a risk-free bond with steady-state interest rate r. Agents are liquidity constrained with respect to private credit markets. Agents differ, however, in their credit limits. Those who are unskilled choose asset holdings a up to a debt limit $\underline{d}^{\rm u} < 0$ units in any given

[5] Hanushek et al. (2003) develop a general equilibrium model of risky schooling and income distribution under risk neutrality, and thus do not consider either intra-generational or intergenerational risk sharing. To focus attention on high-school rather than college schooling, Benabou (2002) studies optimal subsidies with liquidity constraints on human capital and without precautionary savings motives. Seshadri and Yuki (2001) examine both need-based transfers and educational transfers in a model of investment in a child's education when ability is not clearly known. Methodologically, our approach to subsidies is also similar to Li (2002).

[6] A few other recent studies examine college subsidies but do not provide quantitative measures in general equilibrium, or analyze the implications of subsidies for the skill premium or wealth inequality. These include Jacobs and Van Wijnbergin (2002), as well as Wigger and Von Wiezsacker (2001). Our approach is also indirectly related to a larger literature, including Becker and Tomes (1986), Blankenau (1999), Glomm and Ravikumar (1992), Tomes (1981), Loury (1981), Banerjee and Newman (1991), and Galor and Zeira (1993).

period, but no more. To reflect the ability of higher-wage skilled households to borrow at least what low-wage unskilled households can, those who have decided to acquire skills are given extended credit limits. Let the private cost of education be denoted by $\gamma > 0$. The extension of credit limits is kept simple. In the period in which agents attend college, their credit limit is extended by the total private cost of education $\gamma > 0$. Their limit is denoted $\underline{d}^{us}$, where $\underline{d}^{us} \equiv \underline{d}^{u} - \gamma$.[7] Thus, while agents do face constraints on borrowing, these constraints will never prevent a household from being able to finance a college education, and so do not provide any additional rationale for subsidies. Once agents are skilled, they face an extended limit of $\underline{d}^{s}$ units, where $\underline{d}^{s} \leqslant \underline{d}^{us}$.

2.1.1. *Young agents*

Each household periodically produces a new generation ready for college, referred to as 'young.'[8] The arrival of a young (college-ready) member of a family is treated each period as the outcome of a Bernoulli random variable with parameter ρ. The ratio $1/\rho$ is then the average time between college-ready agents within a dynasty, and is observable. Young agents are the representative decision makers for the family/dynasty.[9] Agents are young for only one period, and in subsequent periods are referred to as 'adults.' Our treatment of families thus follows the (related) models of Yaari (1965) and Blanchard (1985).

While planning for college may occur well before the actual enrollment decision is made, with families saving the necessary amount over a long horizon, our results are unlikely to depend on probabilistic arrival of generations for two reasons. First, by allowing households to borrow up to an extended credit limit as discussed above, we provide a serious substitute for savings under perfect foresight. A dynasty that receives a young child can *always* borrow the necessary funds to enroll in college, and pay back the loan afterwards. Given that the rates of interest on loans and savings are identical, individuals who are borrowing to enroll in college are not penalized relative to those who enroll by decumulating their own assets.[10] A second, and perhaps equally important reason is that even if the time horizon for educational savings were known with certainty, the knowledge that a child will eventually be *college-ready* is not known with certainty. In NCES data for the US, over 30% of

[7]See for example, www.ed.gov, for a description of the PLUS loan program, which explicitly allows parents to borrow to finance a dependent child's college education. These loans are not capped in amount.

[8]The process of primary and high school education is not central here and children are modeled as arriving having completed high school.

[9]Heuristically, the parents should be thought of living even after their high-school educated child arrives, as is realistic. However, because we are concerned here with the college education decision of the young agent, we abstract from parents solving another decision problem.

[10]More generally, it is known that in Bewley-style economies, *temporary* shocks can be very effectively smoothed via lending and borrowing. Thus, given our extension of credit limits for college-bound agents, the uncertain timing of the short-term 'expense shock' of investing in college will be easy to smooth. However, long-term shocks, such as the one caused by a failure in college, will remain difficult to smooth. Thus, our economy with relaxed borrowing limits should be seen as a reasonable approximation to an economy in which agents face serious risk in college outcomes, but can buy insurance against the probabilistic timing of college expenditures.

high-school completers are judged to be 'marginally' qualified or 'unqualified' for college.[11] Computationally, an important additional advantage of modeling the arrival of children in this way is that it allows for the elimination of age as a state variable, in a model where the state space already contains the income shock, asset holdings, and education level.

Young agents face the family's current period budget constraint and must decide whether to invest in a risky college education or not, given their wealth, the productivity shock they draw, and the information they possess on their likelihood of success. To avoid exogenously imposing intergenerational persistence, we assume that the young agent draws the labor productivity shock ε from the *unconditional* distribution of productivity. Importantly, we follow Ljungqvist (1993), Caucutt and Kumar (2003), and Fernandez and Rogerson (1995) and assume that college education is a discrete, lumpy investment that cannot be done partially.

2.1.2. Schooling and information on ability

There are two levels of human capital in the model. Those who complete college have human capital level H_s and are referred to as skilled. Those who do not complete college have human capital H_u, and are referred to as unskilled.[12] Skilled agents have higher average productivity than unskilled agents. Skilled workers earn a wage of w_s per unit of labor time, and unskilled agents earn a wage of w_u per unit of labor time.

Young agents choose whether or not to pay the cost and enroll in college. To capture the presence of rapidly increasing opportunity costs following high school completion, unskilled agents born in prior periods do not have the option to acquire education. The explicit private resource cost of obtaining education is denoted by γ, and represents tuition and room and board payments paid by an individual inclusive of all available subsidies. The full cost of education clearly involves the opportunity cost of foregone earnings while in college. Given that students may supply positive hours of labor while also a full time student, we allow them to supply $\upsilon < 1$ units of unskilled labor while in college.

Our environment features adverse selection as Young agents are differentially informed as to the likelihood of successfully completing college. At the beginning of the period, all young agents receive a signal $\mathscr{I}$, drawn from a probability distribution $\zeta(\mathscr{I})$, which determines the information they have on their likelihood of success in college. In reality, some high-school completers do not meet college admission

[11]To the best of our knowledge, there is no study in which education decisions are determined within a calibrated life-cycle model with capital accumulation against which we can closely compare our results. For example, while Caucutt and Kumar (2003) contains a more explicit life-cycle set-up in which children arrive in a deterministic manner, it does not allow for either borrowing or physical capital accumulation, as our study does.

[12]For tractability, we do not model a third class of individuals who may acquire some human capital by enrolling, but not finishing, college. However the gain in earnings for this group appears small (see Hungerford and Solon, 1987; Card and Kreuger, 1992). Additionally, the unemployment rate among college dropouts is much higher than that of college graduates (http://nces.ed.gov/programs/digest/d02/tables/dt380.asp), making the expected return to partial education even lower.

standards, and so will not be able to attend college, and are therefore irrelevant to our study. Let I_f denote the set of households who know they will fail in college with probability one. Conversely, we denote by I_s the set of 'informed' households who know that they will succeed with probability one. Thus, the I_s and I_f groups represent the right and left tails of the distribution of collegiate preparedness, respectively. The signal therefore divides the population of young agents into three groups, $\mathscr{I} = \{I_s, I_f, UI\}$, and is assumed to be i.i.d. across households and over time.

The remainder of households are 'uninformed', denoted *UI*, and are those facing uncertainty with respect to collegiate success. However, within this group, students face uncertainty in college that depends on whether they come from a college-educated or high-school educated household.[13] The likelihood of failure in college for *UI* households is given by $\lambda = \{\lambda_s, \lambda_u\}$, where λ_s and λ_u denote the probability of failure for young agents from skilled and unskilled families, respectively. In order to remain conservative in our approach strictly and avoid generating welfare gains by easing liquidity constraints via subsidies, we treat the probabilities of failure as primitive to the household, and unrelated to tuition levels. Empirical support for this restriction is found in Stinebrickner and Stinebrickner (2003), who study a novel experiment whereby a 100% tuition subsidy was given to students at a Kentucky college. They found that even with full subsidization, failure differentials among income groups persisted, suggesting that 'home-environment' effects are important.[14] Given that the skill premium strongly ties income differentials to education differentials, the law of motion for college preparedness is kept simple: *UI* children of *skilled* parents will be relatively better prepared, with $\lambda = \lambda_s$, with $\lambda_s < \lambda_u$.[15]

Adverse selection is generated in the model by the presence of less-prepared first-generation students enrolling in response to subsidies. The latter turn out to respond much more to changes in subsidies than rich non-first-generation students. In turn, overall dropout rates increase, especially among first-generation households. However, as the government must balance its budget, the cost of subsidy programs arising from adverse selection is explicitly accounted for in our framework.

[13] Allowing greater heterogeneity in preparedness to enrich the representation of adverse selection is possible, but would result in more parameters than calibration targets. Furthermore, such a model would also contain a positive selection effect among well-prepared but poor non-first-generation students. This latter effect will act as an offsetting force to adverse selection. We therefore restrict heterogeneity in ex ante preparedness to these four groups as our calibration will remain disciplined by observable dropout rates among groups well identified in the data.

[14] Stinebrickner and Stinebrickner (2003) also contains additional references documenting income-dependent home-environment effects. They note however that college students from low-income families may still respond to shocks to their families (perhaps parental sickness or unemployment) by returning home. Ex ante, the risk of such events, to the extent that they lead to attrition from college, implies completion risk, just as failure for any other reason would.

[15] For agents who are informed, λ is trivially pinned down. In the case of the informed young agent who will succeed for sure, i.e., $\mathscr{I} = I_s$, it is clear that $\lambda = \lambda_u = \lambda_s = 0$. Similarly, when an agent is sure to fail, i.e., $\mathscr{I} = I_f$, it is clear that $\lambda = \lambda_u = \lambda_s = 1$.

2.1.3. The timing of failure

A critical issue is *when* the uncertainty regarding collegiate success or failure resolves itself. That is, do college dropouts fail immediately and costlessly, or must they wait for a fairly long period in order to learn that they will not be able to successfully complete college? The former entails smaller costs for the individual and the government, relative to the latter. We denote by $\theta_\gamma \in [0,1]$, the fraction of the total available time that is required for *UI* households to find out whether they will succeed or fail. Note that the fraction θ_γ is equivalent to the fraction of the private cost of college that must be paid before uncertainty resolves. If $\theta_\gamma = 1$, households must pay the entire private cost of college to learn whether they will fail or not. Conversely, if $\theta_\gamma = 0$, it is costless to learn about collegiate success, and there is no insurance role for subsidies. In the intermediate cases, for $\theta_\gamma \in (0,1)$, there is a potentially nontrivial trade-off. When θ_γ approaches 0, the insurance benefits of subsidies fall, but so do the costs of financing such programs via distortionary taxes. Conversely, when uncertainty resolves very slowly, with θ_γ approaching 1, the insurance benefits of subsidies grow relatively large, but so do the distortions arising adverse selection and the taxes needed to finance college.

If informed of success, young agents continue their college education by paying the rest of the college cost, $(1-\theta_\gamma)\gamma$. If informed of failure, young agents drop out of college and continue as an unskilled agent in the next period. The timing of payments for college among the uninformed (*UI*) is set to capture the idea that failing students will typically pay only a fraction of the total private cost of college, as they dropout before completing all coursework, as well as to separate the time before, and time after, information revelation without imposing a sub-period in the model. We summarize the timing below.

Sequence of events for young agents

(a) Period t begins. Labor productivity shocks, ε, occur to all agents.
Young agents observe their information set.
They are either informed, (I), and know the outcome of college investment, or are uninformed (*UI*), and do not know with certainty.

(b) Informed young agents who can finish college education successfully ($I = I_s$) *may choose* college by paying γ, the private cost of college. If enrolled, they become skilled in period $t+1$.
Informed agents who will fail with certainty ($I = I_f$) do not enroll in college, and remain unskilled for the rest of their life.

(c) Uninformed (*UI*) agents decide whether or not to enroll in college.
If enrolled, they pay a fraction θ_γ of the cost of college, where $\theta_\gamma \in [0,1]$,
after which the uncertainty of finishing college education, η is resolved.

(d) If the agent is successful, she pays the remaining fraction $1-\theta_\gamma$ of the private cost of college γ, in period $t+1$, and becomes skilled. Otherwise, young agent drops out from college, and stays unskilled.

2.1.4. Subsidies, taxes, and limited liability

Subsidies are modeled as a direct reduction in the private cost of college education. That is, for any given resource cost of attending college, Λ, a subsidy rate $\varphi \in [0,1]$ means that the private cost of attending college is $\gamma = \varphi\Lambda$. In this respect, subsidies in the model are closest to the U.S. public college education system. The subsidy is financed by proportional taxes on labor and capital income, τ_l and τ_c, respectively.[16,17] For all experiments studied here, we use the NCES (1995) estimates to pin down the effective subsidy rate φ under current U.S. subsidy programs.

There are two forms of limited liability in the model, intergenerational limited liability, (*IL*), and collegiate limited liability (*CL*). Given our specification of differential borrowing limits for skilled and unskilled agents, we will require that young agents have intergenerational limited liability for debts incurred by their parents. That is, we assume that parents cannot pass on more than $\underline{d}^{IL}$ units of debt to their college-ready child. A young agent's assets are therefore given by $a^* \equiv \max(\underline{d}^{IL}, a)$. The second limited liability condition applies to the discharge of debts accumulated in college.[18] Limited liability is denoted $\underline{d}^{CL}$, and is set in the benchmark such that agents who fail are relieved of debt in excess of the limit for unskilled agents, $\underline{d}^{u}$.[19]

2.2. The recursive formulation

When an agent is young, she must choose between acquiring a college education or not, and her value function will represent the better of these two options. A young agent's state vector is comprised of four objects, her information level $\mathcal{I} = \{I_s, I_f, UI\}$, her preparedness $\lambda = \{\lambda_s, \lambda_u\}$, her asset level, a, and her current period labor productivity, ε. Her value function is denoted $V^Y(\mathcal{I}, \lambda, a, \varepsilon)$ and must therefore satisfy

$$V^Y(\mathcal{I}, \lambda, a^*, \varepsilon) = \max\{W_C, W_{NC}\}, \qquad (2)$$

where W^C and W^{NC} denote the values from choosing to attend college, or not, respectively. If the agent chooses to attempt college, she will fail to complete college with probability λ, and will attain $V^A(H_u, a^{*\prime}, \varepsilon')$, the value of entering a period as an

[16]We will focus exclusively on the direct subsidies and loan guarantees available to students, and abstract from loan subsidies. Loan subsidies and guarantees have the potential to be important in relaxing liquidity constraints. However, the cost reduction from subsidized loans is miniscule relative to the massive absolute amounts of subsidy directly provided to colleges, public and private.

[17]Winston (1999) finds that the fraction of total costs paid by students is around 40%, but this includes private grants and scholarships as well. As we are interested in explicit public finance of education, and do not model altruistic giving, we hold subsidies fixed at 40%. This is also broadly consistent with the closest relative of this paper, Caucutt and Kumar (2003).

[18]We restrict limited liability to those who fail, as in practice the discharge of student loans requires a formal demonstration of hardship. Student loans also are not dischargeable in a personal bankruptcy.

[19]We emphasize that intergenerational limited liability here is not zero liability, which is the level that prevails in the U.S. at the time of death. A household's debt at the time of a child's college entrance is to be thought of here as the result of consumption smoothing efforts by parents for themselves and their pre-college child.

adult who is unskilled (H_u), with assets and labor productivity $a^{*\prime}$, and ε', respectively. She will succeed in college with probability $(1-\lambda)$, and will then obtain $V^{\mathrm{AColl}}(H_s, a', \varepsilon')$, the value of being a skilled adult (H_s) who just completed college. In the current period she must pay $\theta_\gamma \gamma$ units, and will pay the remainder if she succeeds. An agent's labor endowment while in college is limited by $\upsilon < 1$. Therefore, as seen in Eq. (4), the labor income of a college student is given by $\nu w_u \varepsilon(1-\tau_l)$. If she fails, her asset holdings are protected by the limited liability restriction whereby she is only obligated to pay up to $\underline{d}^{\mathrm{CL}}$. Thus, W_C satisfies the following functional equation:

$$W_C(\mathscr{I}, \lambda, a^*, \varepsilon) = \max_{c,a'} \left\{ \frac{c^{1-\mu}}{1-\mu} + \lambda \beta E V^{\mathrm{A}}(H_u, a^{*\prime}, \varepsilon') + (1-\lambda)\beta E V^{\mathrm{AColl}}(H_s, a', \varepsilon') \right\} \tag{3}$$

subject to

$$c + a' + \theta_\gamma \gamma \leqslant \upsilon w_u \varepsilon(1-\tau_l) + a^*(1+r) - a^* r \tau_c,$$

$$a' \geqslant \underline{d}^{\mathrm{us}}, \; c > 0, \; a^* \equiv \max(\underline{d}^{\mathrm{IL}}, a), \quad a^{*\prime} \equiv \max(\underline{d}^{\mathrm{CL}}, a'), \tag{4}$$

where primes denote next period's variables. If an agent chooses not to go to college, she loses the opportunity to obtain a college education thereafter. She receives utility from current consumption, and becomes an unskilled adult from next period onwards, realizing value $V^{\mathrm{A}}(H_u, a', \varepsilon')$. The value function W_{NC} therefore satisfies:

$$W_{\mathrm{NC}}(\mathscr{I}, \lambda, a^*, \varepsilon) = \max_{c,a'} \left\{ \frac{c^{1-\mu}}{1-\mu} + \beta E V^{\mathrm{A}}(H_u, a', \varepsilon') \right\} \tag{5}$$

subject to

$$c + a' \leqslant w_u \varepsilon(1-\tau_l) + a^*(1+r) - a^* r \tau_c,$$

$$a' \geqslant \underline{d}^{\mathrm{u}}, \quad c > 0, \quad a^* \equiv \max(\underline{d}^{\mathrm{IL}}, a). \tag{6}$$

Notice that the borrowing constraint of the college bound is extended to $\underline{d}^{\mathrm{us}}$ units. Following Cameron and Heckman (1999), this extension reflects the large number of public loan guarantee programs in existence, and rules out credit constraints in college as a direct determinant of outcomes.

A newly skilled adult who has just completed college must pay the remainder of her college costs, $(1-\theta_\gamma)\gamma$, as seen in Eq. (8) below, but is otherwise identical to all other adult agents.[20] In the current period, she obtains utility from consumption. In the following period, she generates a college-ready young agent with probably ρ, whereby she obtains $V^{\mathrm{Y}}(\mathscr{I}, \lambda_s, a^{*\prime}, \varepsilon')$. If she does not generate a child, she will

[20] It is true (by construction) that all agents who successfully complete college education will be able to repay their loans. As a referee has noted, students often carry student loans for long periods of time after graduation. This suggests an additional rationale for our assumption that skilled agents may borrow more than unskilled agents.

instead obtain value $\beta EV^{A}(H_s, a', \varepsilon')$.[21] Therefore, V^{ACoIl}, satisfies the following:

$$V^{ACoIl}(H_s, a, \varepsilon) = \max_{c,a'} \left\{ \frac{c^{1-\mu}}{1-\mu} + \rho\beta EV^{Y}(\mathscr{I}, \lambda_s, a^{*\prime}, \varepsilon') + (1-\rho)\beta EV^{A}(H_s, a', \varepsilon') \right\} \tag{7}$$

subject to

$$c + a' + (1-\theta_\gamma)\gamma \leqslant w_s\varepsilon(1-\tau_l) + a(1+r) - ar\tau_c,$$

$$a' \geqslant \underline{d}^{s}, \quad c > 0, \quad a^{*\prime} \equiv \max(\underline{d}^{IL}, a'). \tag{8}$$

For the remaining agent-types, we have a quite standard consumption/savings problem. We present the formulation under the benchmark case, with all other cases involving straightforward modifications of the parameters and constraints. An adult agent who did not acquire skills in the previous period is described by her human capital level $H = \{H_s, H_u\}$, asset level, a, and current period labor productivity, ε. When an adult is skilled, she maximizes current period utility, knowing that any children she has will be college-ready with probability ρ. This yields a value function, V^{A}, that satisfies:

$$V^{A}(H_s, a, \varepsilon) = \max_{c,a'} \left\{ \frac{c^{1-\mu}}{1-\mu} + \rho\beta EV^{Y}(\mathscr{I}, \lambda_s, a^{*\prime}, \varepsilon') + (1-\rho)\beta EV^{A}(H_s, a', \varepsilon') \right\} \tag{9}$$

subject to

$$c + a' \leqslant w_s\varepsilon(1-\tau_l) + a(1+r) - ar\tau_c,$$

$$a' \geqslant \underline{d}^{s}, \quad c > 0, \quad a^{*\prime} \equiv \max(\underline{d}^{IL}, a'). \tag{10}$$

The preceding can be understood as follows. Adult agents get utility from current consumption, and are replaced with the next generation with probability ρ. If an adult agent is replaced, the young agent becomes the primary decision maker, assumes the assets of the household, must choose whether to attend college or not, and will receive V^{Y}. If an adult agent is not replaced, which occurs with probability $(1-\rho)$, she realizes the value V^{A} in the next period. Unskilled adults are either those who were adults in the previous period, young agents who tried college in the previous period and failed, or young agents who did not attempt college in the previous period. In all three cases, their optimization problem is given above in 9, with only the subscript on human capital H, changing. We turn now to the firm's problem and the government budget constraint.

[21] Note that the joint distribution that the expectation term, $EV^{Y}(\mathscr{I}, \lambda_s, a^{*\prime}, \varepsilon')$, is taken with respect to differs from the distribution with respect to which the second expectation, $EV^{A}(H_s, a', \varepsilon')$, is taken.

2.3. Firms

Let $\mathcal{T}$ denote the aggregate raw labor time devoted to labor supply, given that those who attend college can only work υ units of time:

$$\mathcal{T} = \left[1 - (1-\upsilon)\rho \int_x I(W^{\mathrm{C}}(x) > W^{\mathrm{NC}}(x))\,\mathrm{d}\omega\right]. \tag{11}$$

In the preceding, the term, ω refers to the equilibrium stationary distribution of agents, which will be made explicit below. The indicator function $I = 1$ if $W^{\mathrm{C}}(x) > W^{\mathrm{NC}}(x)$, and $I = 0$ otherwise.

Total raw labor hours supplied are given by $\widehat{N}_{\mathrm{s}}$ and $\widehat{N}_{\mathrm{u}}$ denoting raw skilled and unskilled labor hours, respectively. Wages in the economy will also reflect the presence of any systematic bias in college attendance among those unskilled with high productivity. That is, to the extent that young agents with the high productivity shock are more likely to enroll in college than those with low productivity shocks, the pool of skilled individuals may have higher productivity on average than the unskilled pool. Let $\bar{\varepsilon}^{\mathrm{s}}$ and $\bar{\varepsilon}^{\mathrm{u}}$ denote the average productivities of skilled and unskilled agents, respectively, noting that they might differ from unity due to selection bias. Thus, the effective supply of skilled and unskilled labor are $N_{\mathrm{s}} = \bar{\varepsilon}^{\mathrm{s}}\widehat{N}_{\mathrm{s}}$, and $N_{\mathrm{u}} = \bar{\varepsilon}^{\mathrm{u}}(\mathcal{T} - \widehat{N}_{\mathrm{s}})$, respectively.

There is a neoclassical aggregate production function $F(\cdot)$, which we take to be Cobb–Douglas in physical capital, K, and effective skilled and unskilled labor supply, N_{s} and N_{u}, respectively. Output Y is therefore given as follows:[22]

$$Y = K^{\alpha_1} N_{\mathrm{u}}^{\alpha_2} N_{\mathrm{s}}^{1-\alpha_1-\alpha_2}. \tag{12}$$

Factor and goods markets are competitive, and steady-state pre-tax factor prices (per effective unit) must satisfy the following first-order conditions. For skilled labor we have

$$w_{\mathrm{s}} = F_{\mathrm{s}}(K, N_{\mathrm{s}}, N_{\mathrm{u}}) = (1-\alpha_1-\alpha_2)Y/N_{\mathrm{s}}. \tag{13}$$

For unskilled workers, we have

$$w_{\mathrm{u}} = F_{\mathrm{u}}(K, N_{\mathrm{s}}, N_{\mathrm{u}}) = \alpha_2 Y/N_{\mathrm{u}}. \tag{14}$$

Lastly, the net rental rate of capital satisfies

$$r = F_{\mathrm{K}}(K, N_{\mathrm{s}}, N_{\mathrm{u}}) - \delta = \alpha_1 Y/K - \delta, \tag{15}$$

where δ denotes the rate of depreciation of physical capital.

Given interest rates, and the first-order conditions for the rental rate, we can define a demand function for capital, $K(r)$. We denote demand for skilled labor as $N_{\mathrm{s}}^{\mathrm{d}}$, and demand for unskilled labor by $N_{\mathrm{u}}^{\mathrm{d}}$. Labor supply for skilled and unskilled labor are denoted $N_{\mathrm{s}}^{\mathrm{s}}$ and $N_{\mathrm{u}}^{\mathrm{s}}$, respectively.

[22]While we use it for ease of exposition, the unitary elasticity of substitution between skilled and unskilled labor in our Cobb–Douglas production will not drive results. In the section on robustness, we show that assuming much greater substitutability using a CES production function yields implications for education subsidies that are essentially identical to the Cobb–Douglas case.

2.4. Government

The expenditures for the government arise from the need to service existing debt $(1+r)B$, to pay for government consumption and transfers, G and TR, respectively, to finance subsidy payments to those acquiring human capital, g_{sub}, and to cover both types of limited liability payments. As above, we let young agents' information level be given by $\mathscr{I} = \{I_s, I_f, UI\}$, A be the domain of asset holdings, ε be the support of the distribution of productivity shocks, and $H \in \{H_s, H_u\}$ be the set of human capital levels. $X = \mathscr{I} \times A \times \varepsilon \times H$ denotes the state space for agents, where χ_B will be the Borel σ-algebra on X, and $\omega(Z)$ will be the measure of agents over the state space. The term g_{sub} is found by looking at the flow of young agents into college:

$$g_{sub} = \left[\rho \int_x I(W^C(x) > W^{NC}(x))\,d\omega\right] \frac{\varphi\gamma\theta}{1-\varphi}, \tag{16}$$

where the fraction $\varphi\gamma\theta/(1-\varphi)$ is the cost of providing the subsidy for each student. $I(.)$ is as defined in Eq. (11). L^{IL}, the aggregate expenditure of the government to cover intergenerational limited liability, is found by multiplying the measure of young agents each period in steady state with the conditional mean of their 'excess' debt, $L^{IL} = \rho E(d | d < \underline{d}^{IL})$.

Total limited liability payments for those who accumulate debts for college are given by L^{CL}, and is defined analogously to L^{IL}.

The government raises revenues in two ways, by issuing new bonds B', and by levying taxes on labor and capital income at rates τ_l and τ_c, respectively. The term $\overline{A}r$ refers to interest earned on aggregate asset holdings, conditional on assets being non-negative (since it is only these agents who face capital income taxes). Labor income tax revenues depend on the total wage bill. This is given by the aggregate level of skilled agents, N_s, and unskilled agents, $N_u = (\mathscr{T} - N_s)$, multiplied by the respective wages, $(w_s N_s + w_u N_u)$. In order for budget balance to obtain, we require the following constraint be satisfied:

$$G + TR + (1+r)B + g_{sub} + L^{IL} + L^{CL} = B' + \tau_l(w_s N_s + w_u N_u) + \tau_c(\overline{A}r). \tag{17}$$

3. Equilibrium

We employ the standard recursive stationary equilibrium concept for incomplete-markets models. In particular, we require first that agents optimize, taking time-invariant prices as given.[23] Second, we require that the stationary distribution

[23] As a referee has noted, both college enrollment and the college skill premium have increased substantially over the past half-century. However, addressing the time path of these variables requires a model where the distribution of wealth at each date is a state variable. This is beyond the scope of this paper, but is a key question for future work. In a model without household decision making, Krusell et al. (2000) use the observed increases in skilled labor supply and the skill premium to identify a production function in which technological change is skill-complementary.

generated by household decision rules and income shocks is such that goods markets, asset markets, and labor markets all clear. Lastly, we require the government to maintain a balanced budget.

The individual's asset decision rule for per-capita assets is denoted $a(x)$. The decision rule and the uncertainty of labor income together imply a stochastic process for consumption and asset holdings with an associated transition function $Q(x,Z), \forall Z \in \chi_B$ on the measurable space (X, χ_B). This transition function implies a stationary probability measure $\omega(Z)$ $\forall Z \in \chi_B$ that describes the joint distribution of agents on consumption, asset holdings and credit market status. Given a government debt/output ratio B/Y, and a subsidy rate φ, and birth rate ρ, the following equations will define equilibrium.

Definition 1 (*Equilibrium*). A stationary equilibrium of the model is a profile $\{a(x), N_s, N_u, \omega(Z), \mathscr{T}, r^*, w_s^*, w_u^*, \tau_l^*, \tau_c^*, \overline{\varepsilon_s}, \overline{\varepsilon_u}\}$, that satisfies six conditions:

(i) The decision rule, $a(x)$, is optimal, given r and φ.
(ii) Factor prices satisfy Eq. (13)–(15) above.
(iii) Bond market clearing: $\int_x a(x)\,\mathrm{d}\omega = K(r) + B$.
(iv) Labor markets clear: $N_s = N_s^s = N_s^d$; and $N_u = N_u^s = N_u^d$.[24]
(v) $\omega(Z)$ is a stationary probability measure: $\omega(Z) = \int_x Q(x,Z)\,\mathrm{d}\omega$.
(vi) The government budget constraint, Eq. (17), is satisfied.

4. Welfare

The welfare criterion used here is simply the expected discounted sum of utilities evaluated under the equilibrium stochastic process for consumption. The welfare function also weights all agents equally. It is denoted by W and is given below:

$$W = \int_{Z \in \chi_B} V(Z)\,\mathrm{d}\omega^*(Z). \tag{18}$$

In the above, Z is an element of the σ-algebra χ_B defined earlier on the state space, and indicates current skill, wealth, and productivity levels. $V(.)$ is the agent's value function, and $\omega^*(.)$ is the equilibrium stationary joint density of agents over the state space. This criterion is used in Aiyagari and McGrattan (1998), who provide further justification for its use. In order to compare welfare different subsidy policies, we use a measure of equivalent variation that asks the following question. What proportional increment/decrement in *benchmark* consumption, ξ, will an agent require to make her/him indifferent between being assigned according to probability measure ω^* the benchmark economy and a world with the proposed policy experiment? Thus, $\xi > 0$ implies that a proposed policy improves welfare, and $\xi < 0$,

[24] Note that the aggregate time endowment of the economy is dependent on the relative price of college, as the flow of new college students do not work full-time, but rather $\upsilon < 1$ units. This is accounted for in this definition of labor market clearing, where $N_s + N_u = \mathscr{T}$.

the reverse. Let W^{B} denote welfare in the benchmark economy, and let W^{p} denote welfare under a proposed policy. Given CRRA preferences and the definition of welfare, for risk aversion parameter $\mu > 1$, we have

$$\xi = \left(\frac{W^{\mathrm{p}}}{W^{\mathrm{B}}}\right)^{1/(1-\mu)} - 1. \tag{19}$$

5. Parameterization

5.1. Preferences and income processes

As the model period is one year, we set $\beta = 0.96$. Risk aversion, μ, is set at 2.[25] As mentioned earlier, the persistence of individual income processes is a subject of some debate. We study two cases, which we label the moderate persistence, or 'MP', and high-persistence, or 'HP', cases, respectively. The benchmark MP labor productivity process used is that of Heaton and Lucas (1997), whereby $p_{\mathrm{hh}} = p_{\mathrm{ll}} = 0.75$, and $\varepsilon_{\mathrm{h}} = 1.25$, and $\varepsilon_{\mathrm{l}} = 0.75$. The HP benchmark is set to roughly accord with the evidence of higher persistence documented in Storesletten et al. (1999) and elsewhere, and uses $p_{\mathrm{hh}} = p_{\mathrm{ll}} = 0.95$.[26]

5.2. Production and measurement of hours

Factor shares are set as follows. With our Cobb–Douglas specification, the capital share α_1 is set at 0.36, as in Kydland and Prescott (1982). Labor income shares are set in accordance with the observed skill premium of 1.59 estimated by Autor et al. (1998). The fraction of hours supplied by full-time college equivalents N_{s}, is given by 38.6%, (Autor et al., 1998). The measure reported in the literature (e.g. Autor et al., 1998), is the *fraction of labor hours* supplied by high- and low-skilled agents. However, the total number of hours supplied to the labor market is endogenous, since some agents attend college rather than work full-time. We also do not know hours spent in school or other activities directly. This leads to a distinction between skilled labor hours as a fraction of the total number of hours available to agents, and skilled labor hours as a fraction of labor hours alone. Given Cobb–Douglas production, we can still use the measured ratio of unskilled to skilled labor hours of 1.6, and the observed skill premium of 1.6, to infer identical factor shares with $\alpha_2 = 1 - \alpha_1 - \alpha_2 = 0.32$.[27] The depreciation rate of capital, δ, is set at 0.076,

[25] See, for example, Huggett (1993), Aiyagari and McGrattan (1998), Kubler and Schmedders (2001), and Sheshadri and Yuki (2000).

[26] Kydland (1984) and Mincer, 1991 find that unskilled households face higher risks. However, to isolate the role of subsidies in insuring failure risk, as opposed to subsequent risk on the return from human capital, we restrict attention to identical shock processes across skilled and unskilled agents.

[27] Note that the skill premium, relative labor hours, and factor shares are related by

$$\frac{w_{\mathrm{s}}}{w_{\mathrm{u}}} = \frac{N_{\mathrm{u}}}{N_{\mathrm{s}}}\frac{(1-\alpha_1-\alpha_2)}{\alpha_2}.$$

following Aiyagari (1994). College students can work up to υ units of unskilled labor time while in college. Cuccaro-Alamin and Choy find that median hours worked by full-time students at 4-year schools are approximately 25 h per week, so we set $\upsilon = 0.5$.

5.3. Costs and subsidies

The private cost of college education, γ, is set by noting that the present discounted cost of a public university education was approximately $45,000 (1999 dollars), under current subsidy programs. This is seven percent greater than the average labor earnings for skilled agents of $42,000 (1999 dollars). Therefore, we fix γ, in consumption units, to be $1.07w_{\mathrm{s}}$. In order to fix the subsidy rate, φ, we follow NCES (1995) estimates of total subsidies at U.S. public institutions of 40%. This is also consistent with the share of costs borne by students and their families for U.S. tertiary education, as measured by OECD (2000). We focus exclusively on direct subsidization of education via reductions in tuition, room, and board at public universities, as these constitute the overwhelming majority of total subsidy for public tertiary education. Thus, we set $\varphi = 0.4$. This parameter is the exclusive representation of subsidies in the model, and is varied systematically in our analysis. We explore subsidy rates of up to 100% of the out-of-pocket explicit costs of college. We do not consider subsidizing the opportunity costs of college, or beyond. This restriction in subsidy rates keeps us from confounding costs/benefits arising directly from subsidies with those arising from tax-transfer schemes beyond college.

5.4. Borrowing limits

The borrowing limit for unskilled agents in the benchmark is set at $\underline{d}^{\mathrm{u}} = -2$. This allows agents to borrow up to roughly two times annual income in any period. Recall that agents who are unskilled, but choose to attend college are given a further extension in the credit by exactly the private cost of education, i.e. $\underline{d}^{\mathrm{us}} \equiv \underline{d}^{\mathrm{u}} - \gamma$. This extension in credit is *independent of preparedness*, and represents the complete distortion of lender's decisions by the presence of the government loan guarantee. For simplicity, skilled agents also receive an extension beyond $\underline{d}^{\mathrm{u}}$ equal to the unsubsidized cost of college education, $\gamma/(1-\varphi)$, i.e. $\underline{d}^{\mathrm{s}} = \underline{d}^{\mathrm{u}} - \gamma/(1-\varphi)$.[28,29] For simplicity, the interest rateon loans is the same as the return on savings.

[28]Credit limits are not easily observable, and we wish to avoid finding a positive role for subsidies merely by imposing tight restrictions on asset trade. Therefore, our parameterization of credit limits is deliberately lax relative to the 'no-borrowing' constraints used by Aiyagari (1994), and others, but is in line with limits explored by Huggett (1993).

[29]Incorporating a risk-premium on school loans is of interest, but the presence of loan subsidies in the U.S. makes the simplification more benign. Additionally, a risk-premium would make subsidies even more valuable, while we wish to remain conservative in our approach. We would, in future work, like to motivate this premium via an explicit model of default, such as Ljungqvist et al. (2003).

5.5. Collegiate preparedness

The set of households who we define to be informed is set in accordance with the empirical distribution of extremely well-prepared and extremely poorly prepared students. For this, we employ the NCES Four-Year College Qualification Index.[30] This index is a qualitative measure based on a vector of characteristics, including high school coursework, the 1992 National Education Longitudinal Study Cognitive Test Battery, and SAT or ACT standardized test scores. We define the set of households who are informed of certain success to be those defined by the above index to be 'very highly qualified'.[31] According to the index, this set comprised 13.8% of 1992 U.S. high school graduates. By contrast, the set of households informed of certain failure is taken to be those the index defines to be 'marginally or unqualified', and this set comprised 35.5% of the 1992 U.S. high school graduates.[32] This set of households will be insensitive to subsidies because their test scores and school coursework will deny them admission to essentially any college. Thus, we set $v(\mathscr{I} = I_s) = 0.138$, and $v(\mathscr{I} = I_f) = 0.355$. The remainder of the high school graduating population will be the group of uninformed agents who face a probability of failure conditional on their first-generation status. This implies that $v(\mathscr{I} = UI) = 0.507$. As subsidies are increased, adverse selection will take place whereby less prepared students enter. However, even though the preparedness levels of entering students are elements of a finite set, equilibrium dropout rates change smoothly with subsidies, as households of different wealthlevels respond differently to reductions in the cost of college.

Preparedness levels for uninformed agents, as defined by failure probabilities, are found in the Berkner et al. (2003) data on the relative and absolute completion rates of first-generation, and non-first-generation students. We define a dropout to be the event that after 5 years, the person has no degree, including associates degree or other certificate *and* is no longer enrolled. The failure rate for first-generation students is 42% and is 24% for non-first-generation students. Failure probabilities will be calibrated such the model reproduces the observed skill composition in equilibrium. Autor et al. (1998) estimate that the fraction of college of hours supplied by college-equivalents is 0.386. Because the children of high income/skilled parents typically attend college (see NCES, 1995, COE), we fix the failure probability for non-first-generation young, λ_s, equal to the

[30] Source: http://www.nces.ed.gov/programs/coe/2000/section3/tables/t30_1.html

[31] The NCES definition for this group is: those whose highest value on any of the five criteria would put them among the top 10 percent of 4-year college students (specifically the NELS 1992 graduating seniors who enrolled in 4-year colleges and universities) for that criterion. Minimum values were GPA = 3.7, class rank percentile = 96, NELS test percentile = 97, combined SAT = 1250, composite ACT = 28.

[32] The NCES definition for Marginally or unqualified is: "those who had no value on any criterion that would put them among the top 75 percent of 4-year college students (i.e., all values were in the lowest quartile). In addition, those in vocational programs (according to their high school transcript) were classified as not college qualified."

observed dropout rate of 24%.[33] For first-generation students however, we calibrate the failure probability to match the observed ratio of hours supplied by skilled to that supplied by unskilled labor. While underlying failure rates are unobservable and so are calibrated, *dropout* rates are observable endogenous variables reflecting equilibrium college entrance decisions by the various types of young agents. Therefore, the model's performance in replicating dropout rates will be evaluated. Lastly, the parameter governing the resolution of collegiate uncertainty, θ_γ, is also not definitively observed, but work by Stinebrickner and Stinebrickner (2003) and NCES data together suggest that the median time of failure is roughly two years. We therefore set $\theta_\gamma = 0.5$ in the benchmark case, but also experiment with a broad range of alternative values, ranging from 0.25 (1 year), 0.375 (1.5 years) to 0.75 (3 years).

5.6. Children and government

The waiting time between generations of college-ready students, ρ, is set at 0.0357, reflecting a 28 year average period between college-ready generations. This is set in accordance with the unconditional mean age of a U.S. mother at the time of a child's birth (source: Statistical Abstract of United States, 1999). Lastly, we follow Aiyagari and McGrattan (1998) and set the government consumption/GDP ratio G/Y, at 0.217, and the public debt/GDP ratio B/Y to 0.667. Transfers, TR, are set to zero, for simplicity. The table below documents all model parameters.

Parameters

Parameter	Value	Source
β	0.96	Aiyagari (1994)
μ	2	Aiyagari (1994)
δ	0.076	Aiyagari (1994)
p_{hh}	{0.75,0.95}	Heaton and Lucas (1997)
p_{ll}	{0.75,0.95}	Heaton and Lucas (1997)
ε	{0.75,1.25}	Heaton and Lucas (1997)

[33]Approximately 10 percent of students do not complete high school, and would, in the presence of any selection bias, be at least as likely to fail as the unconditional drop out rate, were they to attempt college. In particular, to the extent that the high school dropout rate is higher among unskilled households, the measured likelihood of failure rate of first generation students is likely to understate the unconditional (on college attendance) probability of failure. In practice, we therefore calibrate λ_u, holding λ_s fixed at 0.24. Alternatively, we could recalibrate ρ to reflect high-school dropouts, but our approach here is more flexible, as it distinguishes between first and second generation students.

α_1	0.36	Kydland and Prescott (1982)
α_2	0.32	CPS (1996)
γ^b	$1.07w_s$	U.S. Dept. of Education (2000) and CPS (1999)
φ	0.4	NCES (1995)
$\underline{d}^u$	-2	Huggett (1993) and Kubler and Schmedders (2000)
$\underline{d}^{us}$	$-2-\gamma^b$	
$\underline{d}^s$	$-2-\gamma/(1-\varphi)$	
$\underline{d}^{IL}$	-2	
$\underline{d}^{CL}$	-2	
λ	$\lambda_s = 0.24, \lambda_u = [0.5350, 0.5975]$	Berkner et al. (2003), calibrated
θ_γ	$\{0.250, 0.375, 0.500, 0.750\}$	
ρ	0.0357	Statistical Abstract of United States (1999)
υ	0.5	Cuccaro-Alamin and Choy (1998)
G/Y	0.217	Aiyagari and McGrattan (1998)
B/Y	0.667	Aiyagari and McGrattan (1998)

Given these parameters, the model produces the aggregate labor endowment, $\mathscr{T}$, skilled and unskilled wages, w^s, and w^u, respectively, an interest rate r, tax rates on capital and labor, τ_c, and τ_l, the skill composition of the labor force N_s and N_u, and consumption and wealth distributions. For tractability and to avoid complicating the results via the effects of differential distortions on factors, we set capital and labor taxes equal, resulting in a single, proportional, factor tax, denoted τ.

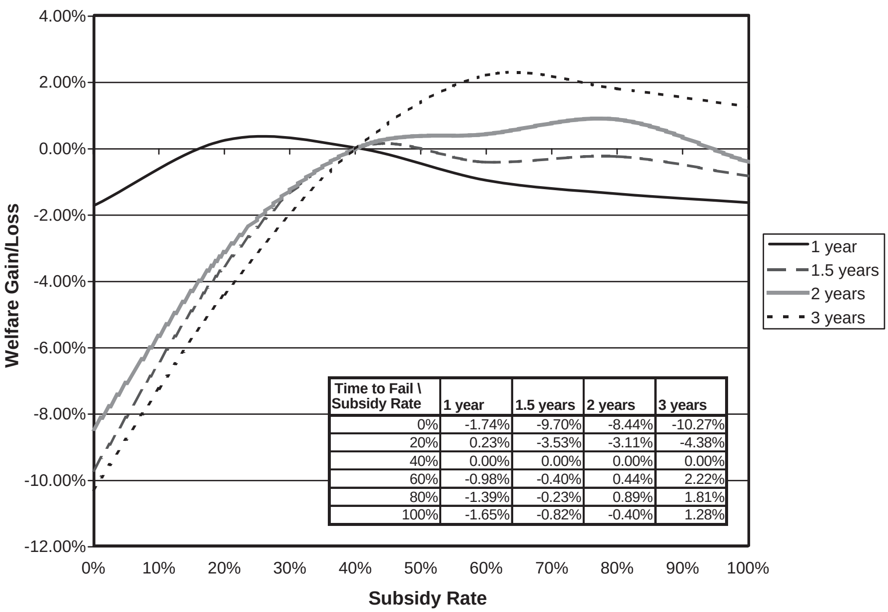
6. Results

Our results are easily summarized. Subsidies to college education improve welfare substantially relative to the fully decentralized allocation. Additionally, subsidies in excess of those observed in the U.S. appear to be welfare improving, given plausible measures of failure risk, although the gains from subsidies beyond rates seen in the U.S. are only modest. We find that increased subsidies lower skill premia, increase interest rates (as precautionary savings fall), lower the variability and inequality of both consumption and wealth, and increase intergenerational income mobility.

Below, we present and discuss the results of our benchmark case in which we (i) set $\theta_\gamma = 0.5$, representing 2 years of enrollment before the outcome of college education is learned; (ii) employ an income shock process with an implied serial correlation of 0.9, which we will refer to as the high-persistence, or 'HP' shock process; (iii) set the elasticity of substitution between skilled and unskilled labor equal at unity, using a production function that is Cobb–Douglas in physical capital, skilled labor, and unskilled labor.

6.1. Subsidies and welfare

We first study the welfare implication of higher education subsidies. In Fig. 2, and in the associated Table 1, we display the welfare gains (losses) of changing the subsidy rate from the current rate (40%) in the HP case, for each of the four levels of θ_γ. These figures show that subsidies increase welfare substantially in all cases. For the benchmark case ($\theta_\gamma = 0.50$, HP income shocks), the loss in welfare amounts to 8.4% in consumption units when the subsidy rate is decreased from 40% to 0%. Additionally, welfare is maximized at a subsidy rate of approximately 80%. The loss of being at the current subsidy rate of 40%, rather than at 80% is however, only 0.8% of consumption. A robust feature of our model is that we observe *large gains from subsidies between 0% and 40%*, but with very little gain or loss beyond 40%. The preceding point provides an explanation for the absence of explicit private insurance contracts against college failure. The gains from trade in such contracts, given observed subsidies, are simply very small.



Time to Fail \ Subsidy Rate	1 year	1.5 years	2 years	3 years
0%	-1.74%	-9.70%	-8.44%	-10.27%
20%	0.23%	-3.53%	-3.11%	-4.38%
40%	0.00%	0.00%	0.00%	0.00%
60%	-0.98%	-0.40%	0.44%	2.22%
80%	-1.39%	-0.23%	0.89%	1.81%
100%	-1.65%	-0.82%	-0.40%	1.28%

Fig. 2. Welfare effects of subsidies.

The welfare gains from subsidies come from three sources. First, the primary force driving our welfare conclusions is that increased subsidies in our model turn out to allow *nearly mean preserving reductions in college risk*. Subsidies increase unskilled wages and lower skilled wages roughly proportionally as more people attend college, narrowing the skill premium and lowering the consequences of collegiate failure. Therefore, the mean return to education, given average failure risk, does not change appreciably across equilibria with different subsidy rates. Thus, subsidies' net impact is to substantially reduce the uncertainty arising from college investment. As will be described in the following sections, consumption *volatility* changes substantially while aggregate output and consumption levels change by much less across different subsidy regimes. The second major force affecting the welfare impact of subsidies is the associated increase in the proportion of 'non-first-generation' students, which implies lower average collegiate risk in the economy. Third, increased subsidies improve efficiency in production by causing the reallocation of investment towards human capital, raising both total output and the interest rate. In fact, for low subsidy rates, increasing subsidies is associated with increases in output even though the physical capital stock falls. A lower capital stock pushes up the return on savings, further aiding consumption smoothing, in very much the same manner as public debt does in the model of Aiyagari and McGrattan (1998).[34] The fall in the steady-state stock of physical capital occurs because households shift their investment to human capital, and because with high subsidy rates (i.e. lower private cost of education), less savings in capital stock is needed to finance college education, or insure against failure in it.

6.2. Prices and quantities

The narrowing of the skill premium provides insurance as in Ljungqvist (1995), and is the primary force in our results. As reported in Table 1, the skill premium goes up from 1.600 in the benchmark economy to 1.946 when subsidies are eliminated. This corresponds to an increase of more than 21% in skill premium. Note that these changes originate in part from a decrease in the unskilled wage rate, from 0.451 to 0.412, corresponding to a decrease of 9.5%.[35] Interestingly, the skill premium as a function of subsidy rates has a shape similar to the welfare function; it is steep for low rates of subsidy, and becomes flat for higher rates. For an example of the latter property, note that the skill premium drops only slightly from 1.600 to 1.597 under a 100% subsidy rate.

In terms of consumption, output, and welfare, eliminating subsidies from the current level implies a drop of 2.79% in average consumption (or from 0.466 units to 0.453 units). A subsidy rate of 100% decreases the average consumption by merely 0.43% from its level in the benchmark subsidy rate. Thus, average consumption also

[34] In all cases considered in this paper, our results understate the true gains from increasing subsidies from zero to the welfare-maximizing level, as there are welfare gains along the transition path, as agents consume capital.

[35] Note that the drops in unskilled wage rate are exacerbated by the unavailability of insurance contracts against income shocks.

Table 1
Allocations

	Subsidy rate	Welfare	*r* (pre-tax)	Tax rate	Skilled wage	Unskilled wage	Skill premium	Fraction of skilled workers	Output	Capital stock	Avg. cons.	Capital/output ratio	Cons./output ratio	CV-C
Time to fail/data	40%		6%	35%	N/A	N/A	1.6	38.00%			N/A	3	0.6	N/A
1 year	0.00%	−1.74%	5.17%	30.85%	0.743	0.442	1.682	36.53%	0.86	2.42	0.467	2.81883	0.545	0.334
1 year	20.00%	0.23%	5.26%	31.11%	0.727	0.448	1.622	37.99%	0.86	2.40	0.471	2.79938	0.549	0.313
1 year	40.00%	0.00%	5.39%	32.00%	0.723	0.446	1.620	37.99%	0.85	2.36	0.469	2.77181	0.551	0.313
1 year	60.00%	−0.98%	5.49%	32.78%	0.718	0.445	1.612	37.99%	0.85	2.33	0.465	2.74999	0.548	0.317
1 year	80.00%	−1.39%	5.57%	33.37%	0.715	0.444	1.611	37.99%	0.85	2.31	0.465	2.73316	0.550	0.322
1 year	100.00%	−1.65%	5.63%	33.90%	0.710	0.445	1.596	37.99%	0.84	2.30	0.466	2.72109	0.551	0.328
1.5 years	0.00%	−9.70%	5.04%	31.00%	0.811	0.410	1.980	33.14%	0.84	2.40	0.453	2.8487	0.538	0.442
1.5 years	20.00%	−3.53%	5.20%	31.00%	0.755	0.434	1.741	36.10%	0.85	2.40	0.465	2.81284	0.545	0.385
1.5 years	40.00%	0.00%	5.35%	31.85%	0.722	0.451	1.600	38.04%	0.86	2.38	0.469	2.77992	0.548	0.341
1.5 years	60.00%	−0.40%	5.39%	32.44%	0.719	0.448	1.605	38.04%	0.85	2.36	0.466	2.77046	0.547	0.339
1.5 years	80.00%	−0.23%	5.46%	33.06%	0.716	0.448	1.599	38.04%	0.85	2.34	0.467	2.75565	0.548	0.337
1.5 years	100.00%	−0.82%	5.49%	33.70%	0.717	0.445	1.611	38.04%	0.85	2.33	0.466	2.74928	0.549	0.347
2 years	0.00%	−8.44%	5.09%	31.20%	0.803	0.412	1.946	33.60%	0.84	2.39	0.453	2.83744	0.538	0.433
2 years	20.00%	−3.11%	5.23%	31.20%	0.752	0.435	1.728	36.34%	0.85	2.39	0.463	2.80702	0.544	0.382
2 years	40.00%	0.00%	5.36%	32.00%	0.721	0.451	1.600	38.04%	0.86	2.38	0.466	2.77885	0.545	0.342
2 years	60.00%	0.44%	5.41%	32.40%	0.714	0.451	1.584	38.04%	0.85	2.36	0.466	2.7671	0.546	0.336
2 years	80.00%	0.89%	5.45%	32.80%	0.715	0.448	1.595	38.04%	0.85	2.35	0.468	2.75862	0.550	0.336
2 years	100.00%	−0.40%	5.51%	33.80%	0.714	0.447	1.597	38.04%	0.85	2.33	0.464	2.74547	0.547	0.346
3 years	0.00%	−10.27%	4.98%	31.00%	0.831	0.402	2.065	32.46%	0.84	2.40	0.451	2.86282	0.537	0.460
3 years	20.00%	−4.38%	5.18%	31.20%	0.767	0.428	1.790	35.39%	0.85	2.40	0.460	2.818	0.541	0.399
3 years	40.00%	0.00%	5.35%	32.00%	0.721	0.451	1.600	37.92%	0.86	2.38	0.466	2.77952	0.544	0.346
3 years	60.00%	2.22%	5.44%	32.20%	0.695	0.462	1.503	39.27%	0.86	2.37	0.469	2.7614	0.547	0.324
3 years	80.00%	1.81%	5.51%	33.10%	0.696	0.459	1.518	39.27%	0.85	2.35	0.467	2.74652	0.547	0.327
3 years	100.00%	1.28%	5.58%	34.10%	0.693	0.458	1.514	39.27%	0.85	2.33	0.465	2.73141	0.546	0.331

has the inverted U-shape of the welfare function. As precautionary savings fall with increased subsidies, interest rates increase monotonically allowing households to insure themselves more effectively by holding capital. For example, (pre-tax) interest rates rise substantially from 5.09% under 0% subsidization to 5.51% under 100% subsidization.

With respect to the volatility of consumption, can agents smooth consumption more easily in an economy with higher subsidy rates? Consumption volatility, as measured by the coefficient of variation (see the column CV-C in Table 1), goes up from 0.34 in the benchmark to 0.43 in the 0% subsidy case corresponding to an increase of 27%. Beyond a subsidy rate of 40% however, consumption smoothing does not improve significantly.

6.3. Educational attainment and long-run mobility

6.3.1. Enrollment and dropout rates

When we examine the behavior of enrollment as a response to different subsidies we obtain a better understanding of the welfare response to subsidies. As subsidies are introduced into the fully decentralized economy, both the enrollment rate and welfare increase. Specifically, as we move from 0% subsidies to 40% in the benchmark case, enrollment rates increase from 54.86% to 64.23% (see Table 2). Such a substantial response of enrollments is reflected in large welfare gains. However, as subsidy rates rise beyond 40%, agents do *not* attend college at a higher rate. Thus, subsidies are not very effective beyond a rate of 40%, and consequently, equilibrium allocations and welfare do not change very much.

We find that the percentage share of first-generation students increases from 58.45% to 60.12% in the pool of total college students (see Table 2). At the same time, the share of non-first-generation students decline monotonically from 41.55% to 39.88%. The change in enrollment documented above reveals clearly that first-generation households are the principal beneficiaries from subsidy programs. Their enrollment rates rise rapidly with subsidies, while the enrollment rate of non-first-generation households responds minimally. The intuition behind this result is that non-first generation households are richer on average, and therefore better able to tolerate collegiate uncertainty. Thus, ex post, it appears that the direct subsidies offered to lower tuition rates benefit some at the expense of others. However, in an ex-ante sense, this insurance offered by subsidies still clearly implies a welfare gain for *all* households, given failure risk.

In terms of success in college, we find that while enrollment rates rise with subsidies, so do dropout rates. As argued above, adverse selection occurs as subsidies encourage *UI* agents, particularly first-generation students, to invest in risky college education. This change in incentives is possibly welfare reducing, if the change in drop out rates among the *UI*, was very large. It is not. The results are given in Table 3. Overall dropout rates rise marginally, in the benchmark case, from 31.87% to 34.78% as subsidies rise from 0% to 100%.[36]

[36]Note that for overinvestment to occur, failure rates would have to be exceptionally high, given the

Table 2
Enrollment rates

	Subsidy rate	Percent of enrolles who are non-first-gen.	Percent of enrollees who are first-gen.	Aggregate enrollment rate
Time to fail/data	40.00%	45.20%	54.80%	66.00%
1 year	0.00%	40.06%	59.94%	61.24%
1 year	20.00%	39.73%	60.27%	64.43%
1 year	40.00%	39.73%	60.27%	64.43%
1 year	60.00%	39.73%	60.27%	64.43%
1 year	80.00%	39.73%	60.27%	64.43%
1 year	100.00%	39.73%	60.27%	64.43%
1.5 years	0.00%	41.61%	58.39%	54.09%
1.5 years	20.00%	40.19%	59.81%	60.08%
1.5 years	40.00%	39.88%	60.12%	64.43%
1.5 years	60.00%	39.88%	60.12%	64.43%
1.5 years	80.00%	39.88%	60.12%	64.43%
1.5 years	100.00%	39.88%	60.12%	64.43%
2 years	0.00%	41.55%	58.45%	54.86%
2 years	20.00%	40.06%	59.94%	60.76%
2 years	40.00%	40.00%	60.00%	64.23%
2 years	60.00%	39.88%	60.12%	64.43%
2 years	80.00%	39.88%	60.12%	64.43%
2 years	100.00%	39.88%	60.12%	64.43%
3 years	0.00%	43.54%	56.45%	50.13%
3 years	20.00%	42.69%	57.31%	56.80%
3 years	40.00%	41.74%	58.26%	62.01%
3 years	60.00%	41.68%	58.32%	64.43%
3 years	80.00%	41.68%	58.32%	64.43%
3 years	100.00%	41.68%	58.32%	64.43%

For non-first-generation students, the model produces fewer dropouts than observed in the data. This is due to the assumption in the model that all I_s agents (i.e. those who are informed to succeed in college) graduate from college with certainty, while in the data, failure rates remain above zero *for all groups*.

Among first-generation students, dropout rates rise by slightly more than among non-first-generation students. Again, in the benchmark case, first-generation dropout rates rise from 42.17% to 45.89% as subsidies rise from 0% to 100%. In contrast, for non-first-generation students, dropout rates for this case rise from 17.09% to just 18.22%. The model also does a very good job matching the data on

(*footnote continued*)
small cost of college relative to the large difference in mean lifetime income between skilled and unskilled households.

Table 3
Dropout rates

	Subsidy rate	First-gen. students	Non-first-gen. students	All young
Time to fail/data	40.00%	45.00%	24.00%	37.00%
1 year	0.00%	45.00%	18.11%	34.23%
1 year	20.00%	46.52%	17.74%	35.08%
1 year	40.00%	46.52%	17.74%	35.08%
1 year	60.00%	46.52%	17.74%	35.08%
1 year	80.00%	46.52%	17.74%	35.08%
1 year	100.00%	46.52%	17.74%	35.08%
1.5 years	0.00%	42.51%	16.74%	31.79%
1.5 years	20.00%	43.55%	18.40%	33.44%
1.5 years	40.00%	45.89%	18.05%	34.78%
1.5 years	60.00%	45.89%	18.05%	34.78%
1.5 years	80.00%	45.89%	18.05%	34.78%
1.5 years	100.00%	45.89%	18.05%	34.78%
2 years	0.00%	42.17%	17.37%	31.87%
2 years	20.00%	43.50%	18.65%	33.55%
2 years	40.00%	45.61%	18.05%	34.59%
2 years	60.00%	45.89%	18.05%	34.78%
2 years	80.00%	45.89%	18.05%	34.78%
2 years	100.00%	45.89%	18.05%	34.78%
3 years	0.00%	37.20%	17.26%	28.52%
3 years	20.00%	38.87%	18.73%	30.27%
3 years	40.00%	40.91%	18.28%	31.46%
3 years	60.00%	42.16%	18.35%	32.23%
3 years	80.00%	42.16%	18.35%	32.23%
3 years	100.00%	42.16%	18.35%	32.23%

the 45% dropout rate of first-generation students in the benchmark case. However, the model underestimates the dropout rate of 24% among non-first-generation students. Thus, there appear to be additional incentives to attend college among poorly prepared rich children.[37]

6.3.2. Intergenerational persistence

We find that income mobility is enhanced by subsidies, often in a quantitatively important way. We measure intergenerational mobility by the correlation of the lifetime incomes of two adjacent generations. The model confirms the widely held view that subsidies can play a useful role in mitigating persistence in incomes across generations.

[37]Assortative matching is one possibility.

Table 4
Inequality and mobility

	Subsidy rate	Gini-cons.	Gini-wealth	CV-cons.	CV-wealth	Inc. mobil.	Wealth mobil.
Time to fail/data	40%	0.30	0.78	N/A	N/A	0.4	N/A
1 year	0.00%	0.18	0.50	0.334	0.956	0.014	0.251
1 year	20.00%	0.17	0.46	0.313	0.881	0.015	0.238
1 year	40.00%	0.17	0.45	0.313	0.869	0.015	0.238
1 year	60.00%	0.17	0.45	0.317	0.901	0.015	0.238
1 year	80.00%	0.18	0.48	0.328	0.961	0.015	0.255
1.5 years	0.00%	0.24	0.66	0.442	1.282	0.150	0.897
1.5 years	20.00%	0.21	0.61	0.385	1.166	0.104	0.903
1.5 years	40.00%	0.19	0.55	0.341	1.036	0.089	0.899
1.5 years	60.00%	0.19	0.54	0.339	1.022	0.089	0.899
1.5 years	80.00%	0.19	0.53	0.337	1.014	0.089	0.903
1.5 years	100.00%	0.19	0.54	0.347	1.059	0.089	0.911
2 years	0.00%	0.24	0.65	0.433	1.262	0.138	0.898
2 years	20.00%	0.21	0.61	0.382	1.165	0.095	0.902
2 years	40.00%	0.19	0.56	0.342	1.041	0.089	0.899
2 years	60.00%	0.19	0.54	0.336	1.022	0.089	0.902
2 years	80.00%	0.19	0.53	0.336	1.015	0.089	0.903
2 years	100.00%	0.19	0.55	0.346	1.066	0.089	0.911
3 years	0.00%	0.25	0.67	0.460	1.299	0.155	0.893
3 years	20.00%	0.22	0.62	0.399	1.201	0.105	0.901
3 years	40.00%	0.19	0.56	0.346	1.050	0.078	0.899
3 years	60.00%	0.18	0.54	0.324	1.020	0.071	0.907
3 years	80.00%	0.18	0.54	0.327	1.024	0.071	0.909
3 years	100.00%	0.18	0.54	0.331	1.041	0.071	0.913

When $\theta_\gamma = 0.50$ with the HP income process, the intergenerational earnings correlation is 0.138 under a subsidy rate of 0%. This correlation quickly falls as subsidy rates rise; with a subsidy rate of 40%, the correlation decreases to 0.089, and does not change when subsidies are increased beyond 40% (see Table 4). Interestingly, our model still substantially understates the income persistence of approximately 0.400 found by Zimmerman (1992), and Solon (1992) in U.S. data. Indeed our results are closer to the estimates of income persistence of the earlier literature, approximately 0.20 (see, for example, Becker and Tomes (1986); Stokey (1996)).[38]

[38]Assortative matching in college may also further stratify college graduates from high school graduates. If so, the risk of failure in college is even greater than documented here, as it may also mean a lower likelihood of finding a college-educated spouse with whom to produce well-prepared children. That is, subsidies, by encouraging young households to take the risk of college, may be even more beneficial in increasing mobility than they are in the present model.

Wealth persistence does not change dramatically with subsidies, but appears to weakly increase. This occurs because precautionary savings fall with increased subsidies, as households will be less affected by the shock of having a *UI* child, relative to an I_s (or I_f) child. That is, the similar expected labor income of households (given parental skill levels), along with subsidies, prevent generations from growing 'too different'. Therefore, the correlation of wealth across generations (which is also the correlation of lifetime wealth across households at a point in time) rises.

6.4. Robustness

We now conduct experiments along three dimensions to evaluate the sensitivity of the results. First, we ask the question: how sensitive are the results to the assumed unitary elasticity of substitution? Second, we assess the importance of the timing of resolution of collegiate uncertainty. Third, the persistence in the shock process of the labor productivity is lowered to investigate its role on allocations and welfare. Our findings turn out to be robust with respect to all three of these dimensions.

We compute equilibria with a higher degree of elasticity of substitution between skilled and unskilled labor when $\theta_\gamma = 0.5$ and income persistence is HP. In all cases, we vary subsidies from 0% to 100%. We study three additional cases for the resolution of uncertainty and 'lumpiness' of college. In particular, we set $\theta_\gamma = \{0.25, 0.375, 0.75\}$, which represents 1, 1.5, and 3 years of enrollment, respectively, before the outcome of college investment is learned. Finally, for each level of lumpiness, we study a substantially lower level of income persistence, MP, with an implied serial correlation of 0.5. All tables and figures are located in Appendix A.

6.4.1. The elasticity of substitution between skilled and unskilled labor

The production technology in (12) implies a unit elasticity of substitution between skilled and unskilled labor. This particular assumption of elasticity of substitution can potentially influence the degree at which the skill premium responds to changes in subsidy rates. Consequently, the implicit insurance provided by lower skill premium may be sensitive to our chosen elasticity. Furthermore, the return to education at the aggregate level is determined by how fast the skill premium decreases with a larger pool of skilled labor, which in turn is determined by the elasticity parameter in the production function. Thus, the efficiency gains of higher subsidy rates may depend importantly on the particular value of elasticity of substitution between skilled and unskilled labor. For this exercise, we assume the following production technology:

$$Y = K^{\alpha_1} N^{1-\alpha_1}, \tag{20}$$

where N denote the 'effective' supply of labor, which is a composite of the labor hours of skilled and unskilled individuals. In particular, N is modeled as a CES aggregate, whereby

$$N = [\varrho N_s^{\kappa} + (1-\varrho) N_u^{\kappa}]^{1/\kappa}. \tag{21}$$

The parameters $\varrho \in (0,1)$, and $\kappa \in (-\infty, 1]$, govern both the substitutability and the skill premium associated with different relative labor supplies. For this production function, the elasticity of substitution is given by $1/(1-\kappa)$. Following Hamermesh (1993), Autor et al. (1998), and Caucutt and Kumar (2003), we set $\kappa = 0.35$, which corresponds to an elasticity of substitution between skilled and unskilled of 1.54. This measure is at the upper end of estimates in the literature, and will therefore provide a good robustness check. The share parameter for the capital stock will remain at 0.36, and we continue with $\theta_\gamma = 0.50$, and HP income shocks, as in the benchmark case. We set $\varrho = 0.5420$, in order to match the observed skill premium under benchmark quantities.

The results demonstrate that the allocation with the higher degree of elasticity of substitution is very close to the benchmark allocation. As shown in Table 5, the welfare is maximized at an approximately 60% subsidy rate. The welfare gains as a function of subsidy rates are high at low subsidy rates, and become flat beyond a 40% subsidy rate. The magnitudes in welfare changes are also very similar; welfare is down by 8.6% in consumption units when the subsidy rate decreases from 40% to 0%. Consistent with the shape of the welfare function, the coefficient of variation in consumption (see the column 'cv-cons.'), which measures the consumption volatility, drops significantly 0.42 to 0.33 when the subsidy rates go up from 0% to 40%, and is relatively constant beyond the 40% subsidy rate. The skill premia follow a similar pattern as well; there is a large increase in the ratio of skilled and unskilled wage rates when the current subsidy rate, 40%, is eliminated altogether.

The second difference is the non-monotonic response of the equilibrium level of capital stock. It increases from 2.32 to 2.36 when subsidies increase from 0% to 40%, and falls to 2.31 with a 100% subsidy rate. The initial increase in the capital stock comes from the higher savings of the larger fraction of skilled workers in the economy. Beyond a 40% subsidy rate, the skill composition does not change whereas the private cost of college education continues to decrease. Thus, agents do not have to use their own savings to enroll in college, and therefore, we observe a drop in the capital stock.

Why are the welfare results robust to substitutability? The major difference in quantities between the equilibria under CES and the Cobb–Douglas benchmark is the significantly larger drop in enrollment in the CES case (from 63.40% to 48.50%) when subsidies are lowered from 60% to 0% (see Table 6). Consequently, the fraction of skilled agents decreases from 38.84% to 31.62% (see Table 5). Many college-ready agents who enroll in college are marginal with respect to their enrollment decision for very low subsidy rates. Thus, when subsidies are lowered to 0%, many college-ready agents choose not to enroll. The wage and output effects of lower enrollment and fewer skilled workers are smaller relative to that occurring with Cobb–Douglas production, because of the higher degree of substitutability between the skilled and unskilled labor in CES production. A smaller fraction of skilled labor

Table 5
Allocations, HP-CES

	Subsidy rate	Welfare	Interest rate (pre-tax)	Tax rate	Skilled wage	Unskilled wage	Skill premium	Fraction of skilled workers	Output	Capital stock	Avg. cons.	Capital/output ratio	Cons./output ratio	CV-C
Time to fail/data	40%		6%	35%	N/A	N/A	1.6	38.00%			N/A	3	0.6	N/A
2 years	0.00%	−8.63%	5.16%	31.40%	0.791	0.409	1.933	31.62%	0.82	2.32	0.443	2.82133	0.539	0.416
2 years	20.00%	−3.85%	5.23%	31.50%	0.749	0.428	1.752	35.02%	0.84	2.35	0.454	2.80598	0.541	0.373
2 years	40.00%	0.00%	5.36%	32.00%	0.712	0.445	1.600	38.20%	0.85	2.36	0.460	2.77886	0.543	0.333
2 years	60.00%	1.38%	5.43%	32.40%	0.703	0.448	1.571	38.84%	0.85	2.34	0.463	2.76294	0.547	0.321
2 years	80.00%	1.28%	5.48%	33.00%	0.702	0.447	1.571	38.85%	0.85	2.33	0.462	2.75209	0.547	0.322
2 years	100.00%	0.31%	5.55%	34.00%	0.700	0.445	1.572	38.80%	0.84	2.31	0.459	2.73769	0.545	0.327

Table 6
Enrollment rates, HP-CES

	Subsidy rate	Percent of enrolles who are non-first-gen.	Percent of enrollees who are first-gen.	Aggregate enrollment rate
Time to fail/data	40.00%	45.20%	54.80%	66.00%
2 years	0.00%	43.33%	56.67%	48.50%
2 years	20.00%	41.89%	58.11%	54.70%
2 years	40.00%	41.17%	58.83%	60.40%
2 years	60.00%	41.17%	58.83%	63.40%
2 years	80.00%	41.17%	58.83%	63.40%
2 years	100.00%	41.17%	58.83%	63.40%

Table 7
Dropout rates, HP-CES

	Subsidy rate	First-gen. students	Non-first-gen. students	All young
Time to fail/data	40.00%	45.00%	24.00%	37.00%
2 years	0.00%	43.94%	16.29%	31.34%
2 years	20.00%	47.42%	16.46%	34.00%
2 years	40.00%	49.57%	16.60%	35.76%
2 years	60.00%	51.21%	16.48%	36.91%
2 years	80.00%	51.21%	16.48%	36.91%
2 years	100.00%	51.21%	16.48%	36.91%

does not significantly increase the marginal product of skilled labor; thus, the decision of not enrolling in college is an *equilibrium* one. In other words, with Cobb–Douglas production, large changes in skill composition would be opposed by large changes in the skill premium, making them less likely to occur in general equilibrium than in the CES case. The relatively larger decrease in enrollment under CES production is associated with a (nearly mean preserving) fall in the skill premium, as occurred in the Cobb–Douglas case. Therefore (again, as in Ljungqvist, 1995), income risk is similar across these economies, leading to similar welfare implications.[39]

With respect to adverse selection, the rise in enrollment rates with increased subsidy rates, especially among the first-generation students, imply a larger drop-out rate as well. Indeed, the drop-out rate among the first-generation students increases from 43.94% to 51.21% when subsidy rates are increased from 0% to 60% (see Table 7). Thus, the adverse selection effects of higher subsides are more pronounced in the CES case. However, these effects are not sufficiently strong to reverse the

[39]As a large portion of the welfare effects present in this model come through price changes arising from enrollment decisions, it is clear that a general equilibrium approach is valuable.

Table 8
Inequality and mobility, HP-CES

	Subsidy rate	Gini-cons.	Gini-wealth	CV-cons.	CV-wealth	Inc. Mobil.	Wealth mobil.
Time to fail/data	40%	0.30	0.78	N/A	N/A	0.4	N/A
2 years	0.00%	0.23	0.63	0.416	1.201	0.233	0.901
2 years	20.00%	0.21	0.59	0.373	1.104	0.192	0.894
2 years	40.00%	0.19	0.55	0.333	1.001	0.154	0.896
2 years	60.00%	0.18	0.53	0.321	0.971	0.133	0.899
2 years	80.00%	0.18	0.53	0.322	0.974	0.133	0.902
2 years	100.00%	0.18	0.54	0.327	0.993	0.133	0.905

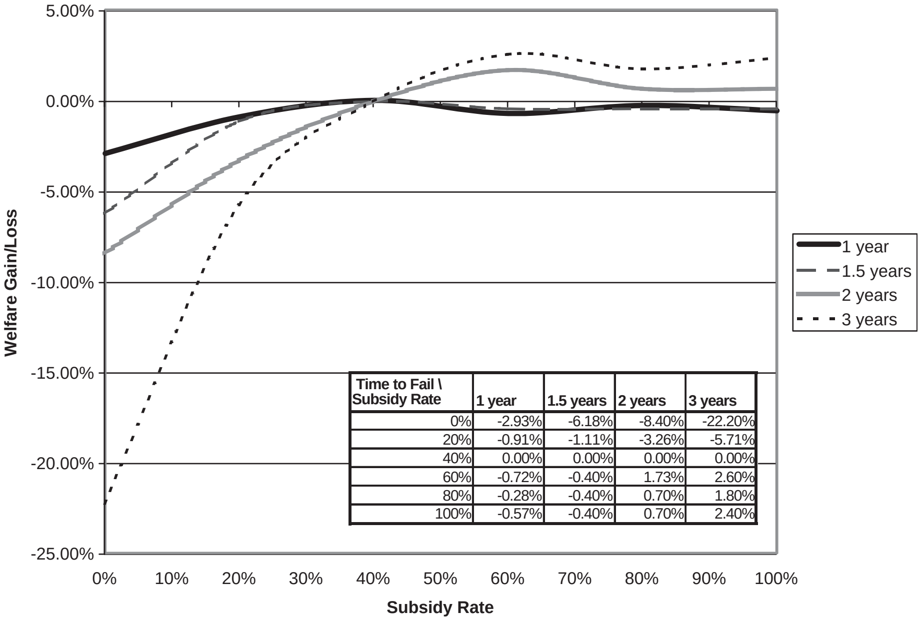


Fig. 3. Welfare effects of subsidies, MP.

welfare gains we observe in the benchmark case with Cobb–Douglas production technology.

Lastly, as Table 8 indicates, the intergenerational earnings correlation is 0.154 with a 40% subsidy rate, and therefore, the model does better in explaining the income persistence we observe in data. Furthermore, eliminating subsidies implies a large decrease in income mobility, as seen in the intergenerational earnings correlation of 0.233.

6.4.2. *The timing of the resolution of collegiate uncertainty*

We now assess the robustness of the results to the resolution of uncertainty in collegiate success, by altering θ_γ. As mentioned earlier, beyond the benchmark

$\theta_\gamma = 0.50$ case, we compute the equilibria of three additional cases, whereby $\theta_\gamma = \{0.25, 0.375, 0.75\}$ (see Table 1.) Figs. 2 and 3 reveal that the later the resolution of uncertainty (i.e. a larger θ_γ), the greater is the welfare gain from moving from very low subsidy rates to high ones. The timing of the resolution of uncertainty in collegiate investment strongly affects the gains from moving away from the observed subsidy rate. When it takes three years to learn about success for uninformed households, (i.e. $\theta_\gamma = 0.75$), the loss relative to current subsidies of eliminating all subsidies is very large, at approximately 10% in the HP case, or \$5000. What is notable is that even when success or failure can be determined in one year, ($\theta_\gamma = 0.25$), we find the welfare response to increased subsidies to be strictly positive and nontrivial for low subsidy rates. In the HP case with $\theta_\gamma = 0.25$, we see a nearly 1.74% gain in consumption, or \$700 per household, as we move from 0% subsidies to 40%. The preceding shows that failure risk remains a dominant force in the welfare effects of subsidies, even when collegiate uncertainty resolves itself fairly early.

To assess the importance of information regarding collegiate success, consider the following exercise. For a fixed subsidy rate, how much do households gain by learning the outcome of college earlier? For the HP case with a subsidy rate of 40%, the gain in consumption is 3.6% if θ_γ decreases from 0.5 to 0.25 (i.e. from 2 years to 1 year). These results also indirectly provide an economic evaluation of the significance of shorter tertiary education programs, such as 2-year programs offered by community colleges, and vocational programs.

As θ_γ is increased, we observe that the enrollment effects of subsidies grow stronger. From Table 2, we see that when θ_γ is increased from 0.25 to 0.375, and then to 0.5 and 0.75, there is a systematic increase in the response of enrollment to subsidies. Under HP income shocks, when subsidies are increased from 0 to 40%, enrollment rises from 61.24% to 64.43% when $\theta_\gamma = 0.25$, from 54.09% to 64.43% when $\theta_\gamma = 0.375$, from 54.86% to 64.43% when $\theta_\gamma = 0.50$, and from 50.13% to 62.01% when $\theta_\gamma = 0.75$. In turn, the skill premium under a subsidy rate of 0% is much higher when uncertainty is resolved in later years. Therefore, subsidies not only make the risk of college palatable, but in so doing cause the skill premium to fall more rapidly in response to subsidies the lumpier is college investment. Welfare, in turn, responds more for a given change in subsidy rates in these cases.

6.4.3. Idiosyncratic income risk

Our calculations also indicate that welfare results are also robust to different degrees of persistence in labor productivity. In particular, the welfare function can be seen to be very similar across cases involving high and more moderate persistence of income shocks. We refer to the latter as the 'MP' case. For the case with $\theta_\gamma = 0.50$, the loss in welfare amounts to 8.4% in consumption in the MP case when the subsidy rate is decreased from 40% to 0% (see Fig. 3 and Table 9). Similarly, welfare is maximized at a subsidy rate of approximately 60%. However, for high levels of θ_γ (i.e. when agents learn the outcome of college education later), the welfare gains/losses are much more striking. For instance, the loss of being at 0% subsidy rate amounts to a drop of 22.20% in consumption when $\theta_\gamma = 0.75$. Note that the skill

Table 9
Allocations, MP

	Subsidy rate	Welfare	Interest rate (pre-tax)	Tax rate	Skilled wage	Unskilled wage	Skill premium	Fraction of skilled workers	Output	Capital stock	Avg. cons.	Capital/output ratio	Cons./output ratio	CV-C
Time to fail/data	40%		6%	35%	N/A	N/A	1.6	38.00%			N/A	3	0.6	N/A
1 year	0.00%	−2.93%	5.48%	30.50%	0.739	0.433	1.709	36.52%	0.84	2.32	0.465	2.753	0.551	0.341
1 year	20.00%	−0.91%	5.55%	31.20%	0.715	0.445	1.605	37.94%	0.85	2.32	0.467	2.738	0.552	0.320
1 year	40.00%	0.00%	5.60%	31.70%	0.714	0.446	1.600	38.00%	0.85	2.31	0.467	2.728	0.552	0.320
1 year	60.00%	−0.72%	5.57%	32.29%	0.713	0.445	1.601	38.00%	0.85	2.31	0.467	2.734	0.552	0.314
1 year	80.00%	−0.28%	5.63%	32.90%	0.711	0.444	1.601	38.01%	0.84	2.30	0.467	2.721	0.553	0.307
1 year	100.00%	−0.57%	5.71%	33.46%	0.709	0.442	1.604	37.96%	0.84	2.28	0.467	2.704	0.555	0.308
1.5 years	0.00%	−6.18%	5.36%	31.00%	0.757	0.427	1.772	35.70%	0.84	2.34	0.458	2.777	0.543	0.377
1.5 years	20.00%	−1.11%	5.52%	31.30%	0.722	0.442	1.635	37.52%	0.85	2.32	0.466	2.744	0.550	0.325
1.5 years	40.00%	0.00%	5.58%	31.60%	0.714	0.446	1.600	38.16%	0.85	2.31	0.469	2.731	0.553	0.321
1.5 years	60.00%	−0.40%	5.64%	32.40%	0.709	0.445	1.591	38.15%	0.84	2.30	0.466	2.719	0.552	0.319
1.5 years	80.00%	−0.40%	5.70%	33.10%	0.706	0.445	1.588	38.20%	0.84	2.28	0.465	2.706	0.552	0.311
1.5 years	100.00%	−0.40%	5.77%	33.90%	0.705	0.443	1.593	38.13%	0.84	2.26	0.464	2.693	0.552	0.300
2 years	0.00%	−8.40%	5.32%	31.30%	0.784	0.414	1.896	34.20%	0.84	2.33	0.454	2.786	0.541	0.400
2 years	20.00%	−3.26%	5.48%	31.60%	0.738	0.434	1.700	36.64%	0.84	2.33	0.461	2.753	0.546	0.347
2 years	40.00%	0.00%	5.59%	31.80%	0.714	0.446	1.600	38.16%	0.85	2.31	0.468	2.729	0.553	0.322
2 years	60.00%	1.73%	5.63%	32.10%	0.705	0.448	1.574	38.40%	0.85	2.30	0.469	2.721	0.554	0.313
2 years	80.00%	0.70%	5.70%	32.80%	0.701	0.448	1.567	38.52%	0.84	2.28	0.468	2.707	0.555	0.306
2 years	100.00%	0.70%	5.81%	34.00%	0.700	0.445	1.573	38.42%	0.84	2.26	0.464	2.685	0.552	0.292
3 years	0.00%	−22.20%	5.00%	32.60%	0.933	0.358	2.607	27.51%	0.80	2.29	0.420	2.857	0.524	0.557
3 years	20.00%	−5.71%	5.41%	32.30%	0.762	0.422	1.806	35.29%	0.84	2.33	0.453	2.768	0.538	0.376
3 years	40.00%	0.00%	5.60%	32.00%	0.714	0.446	1.600	38.17%	0.85	2.31	0.464	2.728	0.549	0.329
3 years	60.00%	2.60%	5.66%	32.20%	0.693	0.455	1.523	39.19%	0.85	2.30	0.469	2.715	0.553	0.302
3 years	80.00%	1.80%	5.80%	33.60%	0.692	0.450	1.538	38.94%	0.84	2.26	0.463	2.687	0.550	0.288
3 years	100.00%	2.40%	5.89%	34.10%	0.686	0.451	1.522	39.20%	0.84	2.24	0.463	2.669	0.551	0.274

Table 10
Enrollment rates, MP

	Subsidy rate	Percent of enrollees who are non-first-gen.	Percent of enrollees who are first-gen.	Aggregate enrollment rate
Time to fail/data	40.00%	45.20%	54.80%	66.00%
1 year	0.00%	38.72%	61.28%	62.11%
1 year	20.00%	39.73%	60.27%	64.43%
1 year	40.00%	39.73%	60.27%	64.43%
1 year	60.00%	39.73%	60.27%	64.43%
1 year	80.00%	39.73%	60.27%	64.43%
1 year	100.00%	39.73%	60.27%	64.43%
1.5 years	0.00%	40.59%	59.41%	58.54%
1.5 years	20.00%	39.79%	60.21%	63.36%
1.5 years	40.00%	39.88%	60.12%	64.43%
1.5 years	60.00%	39.88%	60.12%	64.43%
1.5 years	80.00%	39.88%	60.12%	64.43%
1.5 years	100.00%	39.88%	60.12%	64.43%
2 years	0.00%	41.86%	58.14%	53.99%
2 years	20.00%	40.58%	59.43%	60.46%
2 years	40.00%	40.33%	59.67%	64.43%
2 years	60.00%	40.33%	59.67%	64.43%
2 years	80.00%	40.33%	59.67%	64.43%
2 years	100.00%	40.33%	59.67%	64.43%
3 years	0.00%	43.82%	56.18%	42.98%
3 years	20.00%	43.36%	56.64%	55.25%
3 years	40.00%	41.58%	58.42%	62.49%
3 years	60.00%	41.53%	58.47%	64.43%
3 years	80.00%	41.44%	58.56%	64.33%
3 years	100.00%	41.53%	58.47%	64.43%

premium responds more strongly to changes in the MP case. As θ_γ shrinks, so does the distinction in welfare losses between the HP and MP cases arising from eliminating subsidies altogether. These welfare losses from eliminating subsidies are also order of magnitude smaller than the large losses occurring when $\theta_\gamma = 0.75$, which are 22% and 12%, respectively. Thus, subsidies are more beneficial for income smoothing than they are in the HP case.

As Tables 10 and 11 indicate, enrollment and drop-out rates do not change when we assume a mildly persistent income process (MP) instead of the highly persistent HP income process. This is consistent with the robustness of welfare results to a different income process.

In the MP, $\theta_\gamma = 0.25$ case, as subsidies are increased from 0% to 100%, we see from Table 12 that the correlation of lifetime income across adjacent generations falls very slightly from 0.039 to 0.036. The correlations themselves are very small in

Table 11
Dropout rates, MP

	Subsidy rate	First-gen. students	Non-first-gen. students	All young
Time to fail/data	40.00%	45.00%	24.00%	37.00%
1 year	0.00%	47.21%	17.67%	35.77%
1 year	20.00%	46.52%	17.74%	35.08%
1 year	40.00%	46.52%	17.74%	35.08%
1 year	60.00%	46.52%	17.74%	35.08%
1 year	80.00%	46.52%	17.74%	35.08%
1 year	100.00%	46.52%	17.74%	35.08%
1.5 years	0.00%	44.17%	17.89%	33.50%
1.5 years	20.00%	45.82%	18.01%	34.76%
1.5 years	40.00%	45.89%	18.05%	34.78%
1.5 years	60.00%	45.89%	18.05%	34.78%
1.5 years	80.00%	45.89%	18.05%	34.78%
1.5 years	100.00%	45.89%	18.05%	34.78%
2 years	0.00%	42.15%	17.09%	31.66%
2 years	20.00%	43.82%	18.11%	33.39%
2 years	40.00%	44.97%	18.22%	34.18%
2 years	60.00%	44.97%	18.22%	34.18%
2 years	80.00%	44.97%	18.22%	34.18%
2 years	100.00%	44.97%	18.22%	34.18%
3 years	0.00%	35.60%	17.44%	27.64%
3 years	20.00%	38.58%	17.74%	29.55%
3 years	40.00%	41.80%	18.59%	32.15%
3 years	60.00%	42.31%	18.41%	32.38%
3 years	80.00%	42.31%	18.48%	32.43%
3 years	100.00%	42.31%	18.41%	32.38%

this case. Why? First, recall that income shocks are drawn from the unconditional distribution when an new generation arrives. Second, when $\theta_\gamma = 0.25$, the educational risk is of little consequence, and income risk is effectively smoothed away. Therefore, when $\theta_\gamma = 0.25$, intergenerational income smoothing does not depend importantly on subsidies, as seen in Table 12. This result highlights the fact that the model's internal dynamics are driven by educational lumpiness and uncertainty. As θ_γ rises, the role of subsidies becomes more important for intergenerational mobility. This effect is even more pronounced when $\theta_\gamma = 0.75$, with the correlation falling from 0.242 to 0.0870.

Interestingly, we find that relative to the HP case, lifetime earnings persistence is almost always *lower* than in the MP case. This occurs because with each new generation, shocks are drawn from the unconditional distribution of income shocks. These shocks are, by construction, uncorrelated with those of the previous period. In turn, a highly persistent process has shocks at all dates that are not highly correlated

Table 12
Inequality and mobility, MP

	Subsidy rate	Gini-cons.	Gini-wealth	CV-cons.	CV-wealth	Inc. mobil.	Wealth mobil.
Time to fail/data	40%	0.30	0.78	N/A	N/A	0.4	N/A
1 year	0.00%	0.20	0.56	0.341	1.051	0.039	0.400
1 year	20.00%	0.18	0.56	0.320	1.038	0.036	0.385
1 year	40.00%	0.18	0.55	0.320	1.048	0.036	0.410
1 year	60.00%	0.18	0.55	0.314	0.996	0.036	0.363
1 year	80.00%	0.18	0.52	0.307	0.942	0.036	0.348
1 year	100.00%	0.18	0.51	0.308	0.928	0.036	0.336
1.5 years	0.00%	0.22	0.63	0.377	1.182	0.147	0.939
1.5 years	20.00%	0.19	0.56	0.325	1.032	0.115	0.943
1.5 years	40.00%	0.19	0.56	0.321	1.054	0.105	0.952
1.5 years	60.00%	0.18	0.55	0.319	1.036	0.105	0.949
1.5 years	80.00%	0.18	0.53	0.311	0.988	0.105	0.945
1.5 years	100.00%	0.17	0.50	0.300	0.908	0.105	0.937
2 years	0.00%	0.23	0.63	0.400	1.166	0.185	0.935
2 years	20.00%	0.20	0.58	0.347	1.073	0.124	0.942
2 years	40.00%	0.19	0.55	0.322	1.045	0.100	0.951
2 years	60.00%	0.18	0.54	0.313	1.019	0.100	0.949
2 years	80.00%	0.18	0.53	0.306	0.975	0.100	0.945
2 years	100.00%	0.17	0.49	0.292	0.883	0.100	0.937
3 years	0.00%	0.30	0.73	0.557	1.457	0.242	0.921
3 years	20.00%	0.22	0.60	0.376	1.122	0.167	0.936
3 years	40.00%	0.19	0.57	0.329	1.069	0.097	0.950
3 years	60.00%	0.17	0.53	0.302	1.006	0.087	0.950
3 years	80.00%	0.17	0.50	0.288	0.899	0.087	0.940
3 years	100.00%	0.16	0.47	0.274	0.847	0.087	0.941

with shocks of the previous generation. Thus, higher shock persistence implies lower correlation of income across generations. Nevertheless, subsidies further lower lifetime income correlations across generations. Relatedly, under HP income shocks, we see from Tables 4 and 12 that mobility is less dependent on the time to failure, θ_γ, than in the MP case. In the latter, under 0% subsidies, mobility rises from 0.039 to 0.242, as θ_γ goes from 0.25 to 0.75, while in the HP case persistence rises from 0.014 to 0.155. Thus, subsidies are more effective in the MP case if a higher degree of income mobility is desirable.

Holding subsidy rates fixed, we see that wealth persistence falls appreciably, as we move from the MP case to the HP case. This occurs for the same reason that lifetime earnings correlations across generations falls with persistence. Given that the initial income shock when young is drawn unconditionally, lifetime wealth accumulation, which depends on inherited wealth, but also on lifetime income, becomes less correlated with that of the previous generation as shocks become more persistent.

6.5. Subsidy policy

Perhaps the most interesting finding in this paper is that the optimal subsidy *rate* is still relatively stable across experiments when it takes longer to find out the outcome of college education. As θ_γ approaches 1, the cost of failure grows, thereby increasing the value of a given subsidy rate. However, as θ_γ approaches 1, the cost of a given subsidy program also rises, and demands higher levels of distorting taxes. As a quantitative matter, these effects seem broadly offsetting. That is, optimal subsidy *rates* do not move much, but the gains from subsidies grow much larger as college education becomes more uncertain and lumpy. In spite of this insensitivity, the *gains* from moving to the optimal subsidy rate depend crucially on how early the uncertainty of college investment is resolved.

Our model suggests precisely that it is the gains from increasing subsidies from 0% to 40% that are greatest, with welfare gains flattening out sharply afterwards. That is, employing much higher subsidy rates than 40% appears unlikely to do much damage, while being too stringent with subsidies entails much larger welfare losses. Our results thus help reconcile the observations that subsidy rates vary a great deal across OECD nations, and are typically in excess of 40%. Furthermore, the shape of the welfare function provides insight into why, in the OECD, with its uniformly high subsidies, explicit private insurance against failure appears unavailable: it is unnecessary. It should also be noted that even though the gains from insurance are large under 0% subsidies, adverse selection is likely to impede private provision of such insurance. In particular, highly prepared students will not participate in private insurance programs (a population of 13.8% in our model), while less prepared students (e.g first-generation students) will. A subsidy program, by contrast, forces the participation of all groups.

7. Conclusion

In this paper, we investigated the effect of current tertiary education subsidy policy, especially with respect to its role in encouraging students to invest in risky and lumpy college education. Our results are striking, and easily summarized. We find that not only do existing higher education subsidies improve welfare substantially relative to the fully decentralized equilibrium, but even higher subsidy rates than observed in the U.S. are beneficial. Our results therefore suggest that moderate collegiate subsidies can be justified by appeal to failure risk alone. With respect to prices, quantities, and distribution, we find that increased subsidies lower skill premia, increase interest rates as precautionary savings fall, lower the variability and inequality of both consumption and wealth, and increase intergenerational income mobility.

Our results obtain despite fairly conservative assumptions on the benefits of subsidies and their costs. First, with respect to modeling the benefits of subsidies, as NCES data show, even highly prepared students still fail to complete college at nonzero rates. Rather than allowing such households to face this risk, we posit that they succeed with certainty. Second, by construction no household is credit constrained with respect to financing college education. Third, human capital in our model produces no growth effects, and has only small level effects. With respect to the costs of subsidies, our environment is one where subsidies do lower the average quality of student, and where subsidies are financed via distortionary taxation.

As mentioned earlier, for tractability, we abstract in this paper from allowing partial education to generate positive returns. However, it is likely to be fruitful to eventually relax this restriction. One immediate benefit, for example, would the usefulness of an equilibrium model in gaining insight into the implications of wealth heterogeneity and failure risk for explaining the self-selection of prospective students into 2- and 4-year colleges. This, in turn, would help relate existing subsidies to both observed relative failure rates and the rates of return to each type of education. Such a model could contribute to research on the role of the 2-year or 'community' college in higher education.[40] We hope to pursue this in future work.

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Disclaimer. The views expressed are those of the authors, and do not necessarily represent those of the Federal Reserve Bank of Richmond, or the Federal Reserve System. All errors are ours.

Appendix A. Computation

We compute the stationary equilibrium for the benchmark case as follows. The value functions for the agent's problem are approximated by iterating on the value functions above. Given value functions, we find policy functions for agents by interpolation. With these policy functions in hand, we simulate shock processes and compute quantities by Monte-Carlo integration.[41]

In the benchmark case, we require that the model generate the observed fraction of skilled (and unskilled) agents in equilibrium. Therefore, we fix the birth rate ρ, subsidy rates φ, and preference and technology parameters according to the data. We guess initial average labor productivities $\overline{\varepsilon_s}^0$, and $\overline{\varepsilon_u}^0$, and an initial value for total

[40]See Kane and Rouse (1999).

[41]We use series of length 30,000.

labor supply, $\mathscr{T}^0$, given steady-state education decisions and an initial interest rate r^0. Given interest rates, labor productivities, and our estimates of factor shares, we know *equilibrium* wages w^{s*}, and w^{u*} from the marginal conditions in the representative firm's problem. Given prices, we solve the consumer's problem as described above, and iterate on the non-completion rates for low-preparedness students λ_u to match observed skill composition.[42] We next check capital market clearing, and bisect on interest rates until this market clears. We then check the government's budget constraint, and iterate on tax rates until this constraint is satisfied. We continue until we converge to a vector $(r^*, \tau_c^*, \tau_l^*)$ of interest rates and tax rates, respectively, such that capital markets clear and the government budget constraint is satisfied. Lastly, we check to see if we have a fixed point in $\mathscr{T}$, $\overline{\varepsilon_s}^0$, and $\overline{\varepsilon_u}^0$, and if not, update our guesses.

Outside the benchmark case, we cannot take wages and the skill composition as facts to be matched, but rather must derive them as endogenously determined outcomes. Therefore, we first fix $\mathscr{T}^0$, $\overline{\varepsilon_s}^0$, and $\overline{\varepsilon_u}^0$, an interest rate r^0, and a skilled wage rate w_s^0, and then solve the pair of nonlinear first-order conditions for capital demand k^0, and skilled labor demand, $(N_s^d)^0$. This gives us unskilled labor demand $(N_u^d)^0$, and the *competitive* unskilled wage, w_u^0, conditional on k^0 and $(N_s^d)^0$.[43] The agent's problem can now be solved, as all prices (r^0, w_s^0, w_u^0) are specified. We then iterate on skilled wages w^s until the skilled labor market clears. Given that this market is cleared, we know that the unskilled labor demand is satisfied, and by construction, that the unskilled wage is the competitive one, given the interest rate. We next iterate on the interest rate until the capital market clears. Finally, we iterate on the tax rate, and then the time endowment in order to satisfy the government budget constraint.

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[42]As discussed in Section 6, this calibration is disciplined by observable bounds on non-completion rates in the data.

[43]Equilibrium unskilled labor demand is just 'Time' less skilled labor hours, where 'Time' is aggregate labor hours adjusted for the part-time labor supply of students in college.

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