Effects of Unemployment Insurance Benefits in a Life-Cycle Model*

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Abstract

In a life-cycle model, we calculate the contribution of the generosity of unemployment insurance (UI) on the unemployment rates in Canada and the U.S. The model predicts a 0.81% difference in unemployment rates (one third of the actual difference) in these countries. Social welfare improves if the benefit ratio increases from 45 percent to 55 percent of the working income. Savings drop and average production level declines as a result of higher UI benefit payment.

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1 Introduction

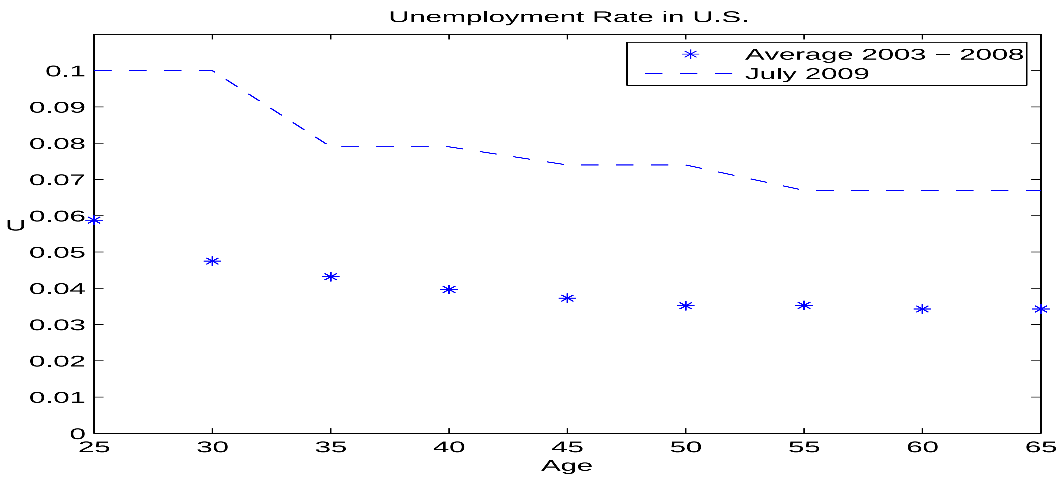
In this paper, we build a life cycle model to analyze the persistent difference in Canada-U.S. unemployment rates from 1982 to 2004. On average, the Canadian unemployment rate was approximately eight percent during this period whereas it is about six percent in the U.S. We assess the contribution of a particular labour market policy to this outcome: the relative generosity of the Unemployment Insurance in Canada.

The publicly-funded unemployment insurance (UI) programmes in the U.S. and Canada were first introduced in 1940s to alleviate income losses of those facing unemployment. Those qualified for the UI benefits do not have to reduce their consumption sharply as the UI provides them a percentage of the lost income, commonly referred to as the replacement rate. Between 1982-2004, Canada had an average replacement rate of 55% whereas the average U.S. replacement rate was about 45%. As in other insurance schemes, the insurance provided by a generous replacement rate comes at the cost of distorted incentives in the form of a lower rate of job acceptance. In particular, as Hansen and İmrohoroğlu (1992) argue, generous replacement rates substantially worsen the moral hazard problem associated with the UI scheme. In this study, we want to quantify the effects of the difference in the generosity of UI across Canada and the U.S. on the unemployment rate in these countries.

When assessing the effectiveness of an insurance scheme, such as the UI, in an incomplete insurance economy, an important issue is the presence of alternative insurance mechanisms that may well interact with the UI scheme. An obvious candidate for such a mechanism is the accumulation of (possibly riskfree) assets as a buffer stock. As Wang and Williamson (2002) argue, agents can accomplish a high degree of consumption smoothing through precautionary saving, rendering the current UI system in the U.S. very close in terms of welfare to an optimal one.

In our life-cycle model, asset accumulation is also influenced by the intertemporal substitution of consumption, given the hump-shaped mean earnings profile of households. Thus, the level of wealth of an individual, in addition to his labour earnings, is age-dependent, implying different degrees of moral hazard (i.e. job acceptance rates) in the labour market for different age groups. Several studies have documented a relatively high unemployment rate among the young households for various countries (see for example Clark and Summers, 1982; Shimer, 1998; Valletta and Hodges, 2005). As shown in Figure 1, we plot both the U.S.'s average unemployment rates from 2003 to 2008 and the unemployment rate in July of 2009. The general trend in the diagram monotonically declines with increase in age.[1] Given the persistence of wage shocks, young households may reject low-paying jobs and wait in the presence of generous UI payments. At the same time, however, low levels of wealth, coupled with borrowing constraints, may encourage young households to avoid unemployed as much as possible. Our model helps us to quantify the above mentioned tension and shed light the extent of the contribution of UI schemes to the high unemployment rates of the young households.

Figure 1: Empirical Age Specific Unemployment Rates



Our numerical results imply that, while a generous UI scheme increases unemployment in a quantitatively important way, it cannot fully account for the difference in unemployment rates in Canada and the U.S. In the baseline case of the U.S. where replacement ratio is 0.45, we find a 2.5 percent of voluntary unemployment rate with a 3.5 percent involuntary unemployment rate in the steady state.[2] When the rate of replacement increases to 0.50, the voluntary unemployment rate changes to 2.84 percent, implying a 0.34 percent increase in the voluntary unemployment rate. When the rate of replacement increases to 0.55, the voluntary unemployment rate changes to 3.31 percent, with another 0.47 increase in the voluntary unemployment rate. Our work is closely related to Andolfatto and Gomme

[1]Shimer (1998) argues that the baby boom generates a large population of young workers in the labor force, and this demographic change is important for the aggregate unemployment. Valletta and Hodges (2005) find that the aging baby-boom generation can account for a significant portion of the decline in the unemployment rate over the past two and a half decades.

[2]In our model, an agent is *involuntarily* unemployed in a period when she receives an exogenous shock separating her from her job. If the unemployed agent rejects a job offer in subsequent periods, she is *voluntarily* unemployed.

(1996) who investigate the effects of the 1972 UI reform on Canada's unemployment rate. They conclude that an increase in the generosity of the UI system leads to a small increase in the unemployment rate of 0.60 percent, but has an important impact on the labor force participation rate. Similarly, our results indicate that the relative generosity of UI system in Canada cannot fully account for the approximately 2.59 percent differential in unemployment rates between Canada and the U.S. Although small, the negative relationship between higher UI benefits and higher unemployment rates does exist, as can be further proved by Joseph and Weitzenblum (2003) and Costain and Reiter (2003). The former shows that reducing replacement ratio always reduces the unemployment rate, while the latter reports that increasing unemployment benefits diminishes employment by decreasing match surplus.

Social welfare, measured in units of consumption, improves overall with an increase of the UI benefits. For the newcomers to the labor market in the U.S., we find that they prefer to give up 0.06 percent of their consumption in order to live in Canada when the rate of replacement increases from 0.45 to 0.50. With the UI replacement ratio increases from 0.45 to 0.55, the welfare gain for U.S. consumers is equivalent to 0.24 percent increase of their consumption.

As argued earlier, risk-free assets, in addition to the UI, provide households insurance against income shocks. One important question is whether households with different wealth levels find generous UI payments desirable. In order to answer this question, we measure the welfare implications of more generous UI systems on 40 years old and 55 years old consumers. The latter age group has the highest level of assets in equilibrium. Measured in the same consumption units, we find that both age groups of consumers prefer to live in Canada where a higher UI benefit is available. Thus, based on the observations on these three groups of consumers, we can conclude that a more generous UI benefit system can improve the social welfare. The fluctuations, regarding the consumption profiles for various age groups, decrease upon an increase in the generosity of the UI benefits. This result is consistent with the analysis of Gruber (1997), where he finds strong consumption-smoothing effects by the UI benefits using the data from the U.S.

As consumers are better insured with more UI benefits, on average they tend to save less. In equilibrium, households' savings are decreasing upon an increase in the generosity of the UI benefits. We find that average savings decrease by 1.1 percent and 4.6 percent respectively when the rate of replacement increases to 0.50 and 0.55 correspondingly, consistent with the simulated results in Gruber (1997) which reports significant negative effects of the UI benefits on savings.

We also find that age-dependent equilibrium unemployment rate has an inverted U shape, with the highest unemployment rate among the rich. That is, when the consumers are asset-rich during their 40s and 50s, they become more likely to voluntarily opt out of job offers and stay voluntarily unemployed. The model underestimates the high levels of unemployment rates among young households. Moreover, a generous UI replacement rate does *not* seem to encourage young households to reject job offers more frequently. At the same time, older (richer) households respond with a high rate of job rejections to increased generosity of UI payments as the marginal value of leisure exceeds of the marginal value of consumption. Hence, our model suggests that moral hazard associated with a UI scheme is more likely to exist among older (richer) households.

We carry out a sensitivity analysis along two dimensions. First, we assume that Canadian consumers are eligible for welfare income assistance in Canada when they are consecutively unemployed for two or more periods. Second, we allow consumers to borrow up to half of their wage income. Both qualitative and quantitative implications are similar to the findings in the benchmark case.

The remainder of the chapter is organized as follows. In section 2, we introduce the model. In section 3, we give a description of equilibrium and the method of computation. Steady state results are discussed in section 4, and robustness analysis is in section 5. In section 6, we provide concluding remarks.

2 The Model Economy

The economy consists of a continuum of overlapping generation consumers, whose working life time j starts from $j = 1$ and ends at $j = J$.

2.1 Consumers

Consumers consume c_j and enjoy l_j at age j. The utility function is represented by $u(c_j, l_j)$, and the future utility is discounted at a rate of $\beta \in (0, 1)$.

Each consumer maximizes

$$E_1\{\sum_{j=1}^{J} \beta^{j-1} u(c_j, l_j) + \gamma\varphi(a_{J+1})\}, \tag{1}$$

where the last term $\varphi(.)$ denotes the utility of the retirement wealth, a_{J+1}, for consumers at age J. We assume that $\gamma > 0$.

At the beginning of each period, consumers receive an idiosyncratic random labor productivity shock $\varepsilon \in \Upsilon$ and face an involuntary unemployment notice $\{\tilde{u}\}$ which is independent across consumers. The labor productivity shock ε is assumed to follow a first-order Markov process, which has seven discrete states. The transition probabilities for these shocks are defined by

$$\Gamma(\varepsilon, \varepsilon') = \Pr(\varepsilon_{j+1} = \varepsilon' | \varepsilon_j = \varepsilon) \text{ for } \varepsilon, \varepsilon' \in \Upsilon \text{ and j } = 1, 2, ..., J. \quad (2)$$

If an unemployed consumer accepts a job offer with a particular productivity ε, he becomes employed, and we write his working state as $\{e\}$. On the other hand, if rejects the job offer, he becomes unemployed voluntarily. This voluntary unemployment and the involuntary unemployment $\{\tilde{u}\}$ constitute the total unemployment in the economy. Denoting the employment state as $\{e\}$ and the involuntary unemployment state as $\{\tilde{u}\}$, the transitional probabilities between them are defined by

$$S(s, s') = \Pr(s_{j+1} = s' | s_j = s) \text{ for } s, s' \in \{e, \tilde{u}\} \text{ and j } = 1, 2, ..., J. \quad (3)$$

By the rule of conditional probability, we can solve for the joint transitional probabilities of the labor productivity shock and the employment status. Writing it as Π, we have

$$\Pi(\pi, \pi') = \Pr(\pi_{j+1} = \pi' | \pi_j = \pi) \text{ for } \pi, \pi' \in \{\Upsilon, e, \tilde{u}\} \text{ and j } = 1, 2, ..., J. \quad (4)$$

2.1.1 A consumer's decision problem

A consumer of age j faces two different employment scenarios. Either he receives a job offer with productivity ε_i for $i = 1, 2, ..., 7$, or he faces an involuntary unemployment notice. In the former case, he could choose to work and supply a fixed amount of working time $n_j \in (0, 1)$, where $n_j + l_j = 1$; or he could turn down the job offer and supply zero amount of working time with $n_j = 0$. This latter option leads to what we call the "voluntary" unemployment.

If the consumer aforementioned chooses to work, he receives an after-tax labor income denoted by $(1 - \tau)n_j w_j^i$, where τ is the tax rate and w_j^i corresponds to age j as well as the labor productivity ε_i. With this specification, we introduce into our model the effects of age on the labor income. The age-dependent wage w_j^i follows a log $AR(1)$ process on productivity level i in addition to the age-dependent factor j.

At the beginning of period j, the consumer at age j has an amount of asset a_j and this asset will earn a gross after-tax return at rate R_j. If the consumer has positive asset holding, he has to pay tax on the interest earned, so that $R_j = (1 + (1 - \tau)(r_j - \delta))$ with asset depreciating at a rate of δ.[3] We assume a single tax rate for both the labor income and the asset income.

Consumers are eligible for the UI benefits only if they have worked in the previous period. For a consumer of age j, he is eligible for the UI benefits when he has worked at age $j - 1$ with $j \geq 2$. Given a replacement ratio of θ, his after-tax UI benefits equal $\theta(1 - \tau)n_{j-1}w^j_{j-1}$, where w^j_{j-1} denotes the wage rate for the jobs with productivity j in the previous period $j - 1$.

If a consumer does not have a job offer and thus becomes unemployed, we assume that he can receive some welfare transfers from the government other than the UI benefits. We denote the welfare assistance by ϕ.

The set of state variables at period j consists of the consumer's age Age_j, his beginning of period asset holding a_j, his previous-period's working status or labor income η_{j-1}, and the current period's job opportunities $\{\varepsilon_i, 0\}$, where 0 means no job offer. At period j for $j = 1, 2, ..., J$, combinations of asset, previous period's working status and current period's job opportunity generate 64 different categories of consumers.

2.1.2 Formulation of consumers' decision problems

Let $v(.)$ denote the solution to the lifetime utility in equation (1). As we have four state variables, v should be written as $v(Age_j, a_j, \eta_{j-1}, \varepsilon_i)$ for those having a job offer and $v(Age_j, a_t, \eta_{j-1}, 0)$ for those who do not have a job offer, for $j = 1, 2, ..., J$. For any new born consumers, they do not have any previous job market experience, thus $\eta_0 = 0$.

During the last period before retirement when $j = J$, consumers of type $(Age_J, a_J, \eta_{J-1}, \varepsilon_i)$ solve the following optimization problem

$$V(Age_J, a_J, \eta_{J-1}, \varepsilon_i) = \max\{V^w_J(a_J, \eta_{J-1}, \varepsilon_i), V^{nw}_J(a_J, \eta_{J-1}, \varepsilon_i)\}, \quad (5)$$

where superscript w indicates 'working' and nw indicates 'not working'. Their corresponding value functions are obtained from accepting the job offer or turning down the job offer respectively. We suppress the state variable Age_J and replace it with a subscript J to the value function.

[3]We performed sensitivity analysis by allowing consumers to borrow up to an exogenous limit. Thus, when $a_j \leq 0$, we have $R_j = (1 + r_j - \delta)$.

At age J, with asset a_J, previous period's working status η_{J-1} and current period's job offer ε_i, consumers accept the job offer whenever the return from working $V_J^w(a_J, \eta_{J-1}, \varepsilon_i)$ outweighes the return from not working $V_J^{nw}(a_J, \eta_{J-1}, \varepsilon_i)$. Consequently, the optimal return from working solves

$$\begin{aligned} V_J^w(a_J, \eta_{J-1}, \varepsilon_i) &= \max_{\{a_{J+1}, c_J\}} \{u(c_J, l_J) + \gamma\varphi(a_{J+1})\} \\ \text{s.t} & \\ c_J + a_{J+1} &= R_J a_J + (1-\tau) n_J w_J^i, \\ a_{J+1} &\geq \max\{0,\ \underline{a}\}, \end{aligned} \tag{6}$$

where $\underline{a}$ is the minimum required savings. We do not have any expected terms on the right hand side of the equation. This is attributed to the fact that consumers will retire at period $J+1$, after which no jobs are available to them. During periods of retirement, consumers rely on their savings to finance their consumption, thus their savings cannot be negative. Put similarly, $a_{J+1} \geq 0$.

The optimal return from not working at period J, $v_J^{nw}(a_J, \eta_{J-1}, \varepsilon_i)$, is the solution to

$$\begin{aligned} V_J^{nw}(a_J, \eta_{J-1}, \varepsilon_i) &= \max_{\{a_{J+1}, c_J\}} \{u(c_J, l_J) + \gamma\varphi(a_{J+1})\} \\ \text{s.t} & \\ c_J + a_{J+1} &= R_J a_J + \begin{cases} \phi & \text{if unemployed at } J-1, \\ \theta(1-\tau) n_{J-1} w_{J-1}^i & \text{with job i at } J-1, \end{cases} \\ a_{J+1} &\geq 0. \end{aligned} \tag{7}$$

Since we have assumed seven states for the labor productivity, there are seven possible values for w_{J-1}^i, each corresponding to a specific labor productivity level.

For consumers who do not have any job offers in the current period and thus have no working decisions to make, they belong to the type of $(Age_J, a_J, \eta_{J-1}, 0)$. If they do not work in previous period of $J-1$, they can receive some welfare assistance from the government. If they have held a job position at $J-1$, they can claim the UI benefits at ratio θ. So they solve

$$\begin{aligned} V_J(a_J, \eta_{J-1}, 0) &= \max_{\{a_{J+1}, c_J\}} \{u(c_J, l_J) + \gamma\varphi(a_{J+1})\} \\ \text{s.t} & \\ c_J + a_{J+1} &= R_J a_J + \begin{cases} \phi & \text{if unemployed at } J-1, \\ \theta(1-\tau) n_{J-1} w_{J-1}^i & \text{with job i at } J-1, \end{cases} \\ a_{J+1} &\geq 0. \end{aligned} \tag{8}$$

Given optimal solutions $V_J(.)$ for consumers of age J, optimal solutions for consumers of younger ages could be solved backwardly, period by period.

Suppose that there is a consumer at age $J-1$ with asset a_{J-1} who has worked in previous period of $J-2$, and has a job offer with productivity ε_i. Let's denote his optimal value function as $V_{J-1}(a_{J-1}, \eta_{J-2}, \varepsilon_i)$, then

$$V_{J-1}(a_{J-1}, \eta_{J-2}, \varepsilon_i) = \max\{V_{J-1}^{w}(a_{J-1}, \eta_{J-2}, \varepsilon_i), V_{J-1}^{nw}(a_{J-1}, \eta_{J-2}, \varepsilon_i)\}, \tag{9}$$

where, as usual, superscripts w and nw mean 'working' and 'not working' respectively.

Solution to $V_{J-1}^{w}(a_{J-1}, \eta_{J-2}, \varepsilon_i)$ can be obtained from

$$\begin{aligned} V_{J-1}^{w} &= \max_{\{a_J, c_{J-1}\}} \{u(c_{J-1}, l_{J-1}) + \beta \sum_{\pi'} \Pi(\varepsilon_i, \pi') V_J(a_J, \eta_{J-1}, \pi')\} \\ \text{s.t} & \\ c_{J-1} + a_J &= R_{J-1} a_{J-1} + (1-\tau) n_{J-1} w_{J-1}^{i}, \\ a_J &\geq \underline{a}, \end{aligned} \tag{10}$$

where π' could be any job offers with productivity ε_i or no job offer thus can be written as 0. Here savings can be negative if $\underline{a} < 0$, i.e., consumers can borrow to finance their consumption, which is not allowed when consumers are in their last period right before retirement.

Similarly, solution to $V_{J-1}^{nw}(a_{J-1}, \eta_{J-2}, \varepsilon_i)$ can be described in the same way, except that we need to take into account whether this consumer worked in the previous period of $J-2$ or not. If he has worked, he is eligible for the UI benefits; otherwise, he can receive some welfare assistance from the government other than the UI benefits. We thus have

$$\begin{aligned} V_{J-1}^{nw} &= \max_{\{a_J, c_{J-1}\}} \{u(c_{J-1}, l_{J-1}) + \beta \sum_{\pi'} \Pi(\varepsilon_i, \pi') V_J(a_J, \eta_{J-1}, \pi')\} \\ \text{s.t} & \\ c_{J-1} + a_J &= R_{J-1} a_{J-1} + \begin{cases} \phi \ \text{ if unemployed at } J-2, \\ \theta(1-\tau) n_{J-2} w_{J-2}^{i} \ \text{ with job i at } J-2, \end{cases} \\ a_J &\geq \underline{a}. \end{aligned} \tag{11}$$

For a consumer of type $(a_{J-1}, \eta_{J-2}, 0)$, optimal solution $V_{J-1}(a_{J-1}, \eta_{J-2}, 0)$ is very similar to the V_{J-1}^{nw}. The only difference is the expectations on the right hand of the maximization (11).

For consumers aged from $j = 2$ to $j = J-2$, their utility optimization problems could all be solved in the same way as those for consumers of age $J-1$. Because we have obtained the optimal value functions for consumers at age J, solutions to earlier periods can make use of the optimal value functions of later periods.

For consumers entering the job market at age, $j = 1$, we assume that they do not have any asset holdings, i.e., $a_1 = 0$. The state vector can be described as $(Age_1, a_1, \varepsilon_i)$ with job opportunity ε_i and $(Age_1, a_1, 0)$ without job opportunity. Since period $j = 1$ is the first time that consumers enter the labor market, they do not have any previous job market experiences, and thus they do not have working decisions to make based on the availability of the UI benefits. If they are offered a job with productivity ε_i, they will accept it. Otherwise, they become involuntarily unemployed.

Given a job offer with productivity ε_i, a consumer's utility optimization problem can be expressed as

$$\begin{aligned} V_1(a_1, \varepsilon_i) &= \max_{\{a_2, c_1\}} \{u(c_1, l_1) + \beta \sum_{\pi'} \Pi(\varepsilon_i, \pi') V_2(a_2, \eta_1, \pi')\} \\ \text{s.t} & \\ c_1 + a_2 &= R_1 a_1 + (1-\tau) n_1 w_1^i, \\ a_2 &\geq \underline{a}, \end{aligned} \tag{12}$$

with $V_2(a_2, \eta_1, \pi')$ solved at time $j = 2$.

If a consumer at time $j = 1$ does not have a job offer, his problem is given by

$$\begin{aligned} V_1(a_1, 0) &= \max_{\{a_2, c_1\}} \{u(c_1, l_1) + \beta \sum_{\pi'} \Pi(0, \pi') V_2(a_2, 0, \pi')\} \\ \text{s.t} & \\ c_1 + a_2 &= R_1 a_1 + \phi, \\ a_2 &\geq \underline{a}. \end{aligned} \tag{13}$$

Since he does not have a job offer, the government will provide him ϕ units of the consumption good as welfare assistance.

2.2 Firms

There is a competitive sector of profit-maximizing firms hiring labor N_t and renting capital K_t to produce output. The technology for production is assumed to be Cobb-Douglas

$$Y_t = F(K_t, N_t), \tag{14}$$

where K_t is the aggregate capital, and N_t is the aggregate labor in efficiency unit. Production function $F(.,.)$ is a constant returns to scale technology and satisfies Inada conditions.

Firms solve the following static problem in a period t,

$$\max_{\{K_t, N_t\}} F(K_t, N_t) - r_t K_t - w_t N_t. \tag{15}$$

Since there are no aggregate shocks in our model, factor prices are constant in the steady state, and therefore, we can drop the time-subscript. The first-order conditions of the above problem yield the following functions for the wage rate and rental rate of capital:

$$\begin{aligned} r &= \frac{\partial F(K,N)}{\partial K}, \\ w &= \frac{\partial F(K,N)}{\partial N}. \end{aligned} \tag{16}$$

Here the wage rate w is the average wage that firms could offer based on its profit optimization results. It is the mean of the wages for consumers of different ages under various productivity shocks ε_i.

2.3 Government

Government serves as the intermediary on the UI system. It collects income taxes to finance the UI benefits and the welfare assistance. We assume that the government operates without operation cost thus the total taxes collected equal the total benefits handed out. In essence, the government runs a balanced budget every period.

Let T denote total taxes collected, which is the sum of two parts. The first part is the tax imposed on consumers' labor income and the second part is the tax on capital returns. Specifically, T can be written as

$$T = \int_a \sum_{j=1}^{J} \sum_{\varepsilon_i} \sum_{\nu_j = i} \tau n_j w_j^i + \int_{a>0} \sum_{j=1}^{J} \sum_{\varepsilon_i} \tau (r - \delta) a, \tag{17}$$

where we have applied $\nu_j = i$ to represent the circumstances that consumers accept the job offer with productivity ε_i.[4]

Let B denote the total benefits, which includes the after-tax UI benefits and the welfare assistance. To calculate B, we fist calculate B_0 for those who do not have a job offer in

[4] We include the UI tax in the formation of benefit B which includes both the welfare payment and the after-tax UI benefits payments.

the current period

$$B_0 = \int_a \sum_{j=1}^{J} \sum_{\tilde{u}} [\sum_{\nu_{j-1}=0} \phi + \sum_{i=1}^{7} \sum_{\nu_{j-1}=i} \theta(1-\tau) n_{j-1} w_{j-1}^i] \tag{18}$$

The first part of B_0 is the welfare assistance to those who have not worked in the previous period $j-1$ (thus $\nu_{j-1} = 0$) and who do not have job offers (thus $\tilde{u}$) in the current period j. These consumers receive the welfare assistance ϕ from the government. The second part is the UI benefits to those who have worked in the previous period (thus $\nu_{j-1} = i$ for $i = 1, 2, ..., 7$) but do not have job offer (thus $\tilde{u}$) in the current period. Here w_{j-1}^i is the previous period's labor income with job productivity i.

Next we calculate the benefits handed out to those who have a job offer ε_m in the current period, where $m = 1, 2, ..., 7$ is the index for job productivity. The formula for B_m is

$$B_m = \int_a \sum_{j=1}^{J} \sum_{\nu_j=0} [\sum_{\nu_{j-1}=0} \phi + \sum_{i=1}^{7} \sum_{\nu_{j-1}=\varepsilon_i} \theta(1-\tau) n_{j-1} w_{j-1}^i]. \tag{19}$$

The first part for B_m is the welfare assistance for those who have not worked in the previous period (thus $\nu_{j-1} = 0$) and who reject the job offer ε_m (thus $\nu_j = 0$) in the current period j. Since they have not worked in the previous period $j-1$, they can only receive welfare assistance ϕ after turning down the job offer ε_m. The second part is the UI benefits at ratio θ to those who have worked in the previous period (thus $\nu_{j-1} = \varepsilon_i$ for $i = 1, 2, ..., 7$) and who reject the job offer ε_m at period j. They can receive the after-tax UI benefits at a ratio θ of the previous period's labor income.

The total government expense is given by

$$B = B_0 + \sum_{m=1}^{7} B_m. \tag{20}$$

The equivalence of the total taxes T and the total benefits B can be achieved by manipulation of the tax rate τ. In equilibrium, a tax rate τ^* guarantees $T = B$.

2.4 Equilibrium

Definition 1 *The steady state equilibrium for this economy consists of a set of the value functions $\{V_j\}_{j=1}^J$, the decision rules $\{a_{j+1}, c_j, \nu_j\}_{j=1}^J$, the factor prices $\{r, w\}$, a government tax rate τ^*, and a measure of the consumers ω such that*

1. *Under the factor prices* $\{r, w\}$ *and the government tax* τ*, the functions* $\{V_j, a_{j+1}, c_j, \nu_j\}_{j=1}^{J}$ *solve consumers' decision problems.*

2. *Factor prices are given by marginal productivities, i.e.,* $r = F_K$ *and* $w = F_N$*.*

3. *Factor markets clear:*

   (a) $K^D = \int_a \sum_{j=1}^{J} a_{j+1} d\omega,$

   (b) $N^D = \int_a \sum_{j=1}^{J} \sum_{i=1}^{7} n_j \varepsilon_i d\omega.$

4. *The distribution measure* ω *is stationary.*

5. *The tax rate* τ^* *balances the government budget constraint.*

6. *By virtue of Walras law, the aggregate resource constraint of the economy is satisfied, i.e.,* $C + K' = F(K, N) + (1 - \delta)K$*.*

3 Calibration of the model

The model period is set to a quarter of a calendar year. The related discount factor, β, is set to 0.99, implying an annual discount rate of approximately four percent.

The utility function, u, has the form:

$$u(c_j, l_j) = \frac{c_j^{1-\sigma} - 1}{1 - \sigma} + \lambda l_j, \text{ for } j = 1, 2, ..., J, \tag{21}$$

where λ_j is the age-independent weight assigned to leisure for consumers of age j.

The risk aversion parameter σ is set to 2. The weight assigned to leisure in the utility valuation, λ, is calibrated to fit the 6 percent unemployment rate in the postwar U.S. economy. In the steady state, we find that $\lambda = 0.12$. We further assume that labor is indivisible, thus $n_j = \overline{n}$ for all $j = 1, 2, ..., J$. Based on the Establishment Survey Data (available from the Federal Reserve Bank of St. Louis), we set $\overline{n} = 0.33$. That is, consumers supply one-third of their time to work.

The retirement felicity function is assumed to take the following form

$$\varphi(a_{J+1}) = \frac{a_{J+1}^{1-\varrho} - 1}{1 - \varrho}, \tag{22}$$

where we have assumed that the retirement wealth is given by the final period's asset savings a_{J+1}.

The parameter ϱ which governs the consumer preference for retirement wealth, a_{J+1}, is set to 2.0. The weight assigned to the retirement wealth function, γ, is calibrated so that the ratio of the retirement savings to the average labor income in the model is equivalent to the ratio of the median retirement savings to the median income in year 2004 for the U.S. households.[5] We find that $\gamma \doteq 7.0$.

The production technology is assumed to be Cobb-Douglas, that is, $F(K,N) = K^{\alpha}N^{1-\alpha}$. Capital share in production, α, is set to 0.36. The capital depreciation parameter δ is set to 0.025, corresponding to a 10 percent annual depreciations.

The (log) wage process w_j^i is assumed to be

$$\ln w_j^i = \mu_j + x_j^i, \;\; j = 1, 2, ..., J; \;\; i = 1, 2, ..., 7, \tag{23}$$

where μ_j is the age-dependent wage income, x_j^i is the first-order labor productivity shock. This second productivity shock x_j^i at each age j equals the labor productivity shock ε_i for $i = 1, 2, ..., 7$.

The age-specific wage income $\{\mu_j\}_{j=1}^{J}$ is equal to that in Hansen (1993). As the original data are in annual frequency, a linear interpolation is applied to convert them into a higher frequency of quarterly data. As consumers get older, their age-dependent average wage income increases initially, peaks when they are between 50 to 55 years old, then decreases until they retire.

The first-order Markov process for the labor productivity x_j^i is discretized with seven states by Tauchen (1986)'s method, based on the following specification

$$x_j^i = \rho x_{j-1}^i + m_i, 0 < \rho < 1, i \geq 2, \tag{24}$$

where the error term $m_i \sim$ i.i.d $N(0, \sigma_m^2)$. To distinguish the newcomers to the labor market from those already in the labor market, we assume that $x_0^i = 0$ and $m_0 \sim$ i.i.d $N(0, \sigma_o^2)$.[6]

We assume that consumers enter the labor market at the age of 25 and retire at the age of 65. The set of parameters for the idiosyncratic productivity shock is calibrated as follows. The variance of the initial shock σ_o^2 is set to allow the model to match the variance of the log income for the youngest consumers from the PSID data. The persistent parameter ρ is used to generate the linear life-cycle growth of variance in the log income. The volatility parameter σ_m is calibrated to capture the increase in the log income variance over the life-cycle, from newcomers into the labor market to the oldest workers. This method is

[5] See http://www.census.gov/prod/2006pubs/p60-231.pdf.

[6] Thus x_1^i has a different transitional probability from those x_j^i when $j \geq 2$. Taking this into account, we discretize x_1^i separately using the same method but with a different level of volatility σ_o.

similar to that in Athreya (2008), and the targeted values are obtained from Storesletten, Telmer and Yaron (2004). We find $\sigma_o = 0.53$, $\rho = 0.98$, and $\sigma_m = 0.12$. These estimates are close to those in Storesletten, Telmer and Yaron (2004).

By Tauchen's method, transitional probability matrices Γ are calculated for both the persistent labor productivity x_j^i for $j > 2$ and the noise productivity x_1^i. The transitional probability matrix, S, for employment and unemployment state, is obtained as follows. Based on observations in Obiols-Homs (2003), average unemployment spells in the U.S. are 15 weeks. This is used to calculate the transitional probabilities from unemployment to unemployment and unemployment to employment. To obtain the other probabilities from employment to both unemployment and employment, we need to find out the stationary involuntary unemployment opportunity $p(\tilde{u})$, which has a lower bound of zero and an upper bound of 6 percent for the postwar U.S. economy. Starting from zero to six percent, we apply a grid search method and try to match the distribution of the unemployment spells from our steady state results to those found in postwar U.S. economy.[7] Finally, we find that $p(\tilde{u}) = 0.035$.

Given the stationary involuntary unemployment rate $p(\tilde{u})$, transitional probability matrix S can be obtained by method in Hansen and İmrohoroğlu (1992). The resulting transitional probability Π governing both labor productivity and employment status is solved by rules of conditional probabilities. As we have seven states for the $AR(1)$ labor productivity, the resulting matrix Π has dimension one larger than the number of idiosyncratic shocks. We assume that the lower bound of asset is zero, i.e., $\underline{a} = 0$ in the benchmark economy.

The replacement ratio, θ, is set to 0.45 for the benchmark economy which corresponds to the average replacement rate in the U.S.. In our experiments, we compute equilibrium with $\theta = \{0.50, 0.55\}$ respectively which correspond to the replacement ratio for the Canadian economy. The welfare payment ϕ is calibrated as follows. First, we find the ratio of the annual minimum wage to the average income assistance in the U.S., which is approximately equal to 2.44. Then we divide the minimum age-dependent wage in the model by this ratio. The resulting statistic is the constant welfare assistance ϕ.

[7] See Table A-9 in BLS website http://www.bls.gov/news.release/empsit.t09.htm.

3.1 Examples of optimal policies

In our model, there are forty years of admissible working time for each consumer. At any specific age j, consumers belong to one of the 64 categories defined by various combinations of three state variables $(a_j, \eta_{j-1}, \varepsilon_i)$. Savings, consumptions as well as working decisions are thus different among these categories.

As a general rule, saving functions are always nondecreasing. If consumers do not have a job in the current period and have to rely on welfare assistance from the government, $(a, 0, 0)$, they need to dissave to finance their consumption. If they have worked during the previous period but do not have a job offer in the current period $(a_j, \eta_{j-1}, 0)$, they can receive the UI benefits at the ratio of θ. Depending on the values of η_{j-1}, they might dissave at all the time or save first and then dissave later. Consumers who have worked in the previous period with $\eta_{j-1} = \varepsilon_j$ and have a job offer ε_i in the current period belong to type $(a, \varepsilon_j, \varepsilon_i)$. They have two saving decisions to make. The first is the savings made if they choose to work with the job offer ε_i, and the second is the savings if they choose to receive the UI benefits at a rate of θ of the previous period's labor income. The dividing point between these two parts is determined by the *asset cutoff point*, which is obtained from comparison of the values from working and the values from not working.

4 Steady State Results

Under the benchmark case (which corresponds to the U.S. economy) with $\theta = 0.45$, a 5 percent increase in this ratio increases the voluntary unemployment rate by 0.34 percent. That is, the total unemployment rate is found to be 6.34 percent. A further 5 percent increase in θ to 0.55 generates an additional 0.47 percent increase in the voluntary unemployment rate, leading the overall unemployment rate to 6.81 percent. In order to replicate the average Canadian unemployment rate of 8.59 percent, we find that the rate of replacement θ must be around 0.65 in equilibrium. The overall social welfare improves with a more generous UI benefit system. At the same time, a more generous UI system lowers the savings since it provides better insurance to consumers against the unemployment shocks.

In the subsections below, we focus on the implications of the generosity of the UI system on three aspects. The first is the unemployment rate, followed by the social welfare, then the output as well as the savings.

4.1 Implications on Unemployment

With a calibrated level of 3.5 percent stationary involuntary unemployment rate and a 6 percent total unemployment rate for the postwar U.S. economy, voluntary unemployment rate in the U.S. is found to be at 2.5 percent. We report the equilibrium results in Table 1.

Table 1: Taxes and Labor Market Outcomes

	$\theta = 0.45$(benchmark)	$\theta = 0.50$	$\theta = 0.55$
Tax rate τ%	1.74	1.94	2.22
Unemployment rate $\overline{U}$%	6.00	6.34	6.81
Involuntary unemployment rate	3.50	3.50	3.50
Voluntary unemployment rate	2.50	2.84	3.31
Percent of unemployment spells			
1-period-unemployed	85.03	85.61	86.40
2-period-unemployed	10.02	9.64	9.09
3-period-unemployed	1.97	1.88	1.81

First, a more generous UI benefit system has to be financed with it a higher tax rate in the steady state. Since there are more voluntarily unemployed consumers and they are to receive more UI benefits, the government has to impose a higher tax in order to balance its budget in equilibrium. Furthermore, consumers also hold a smaller level of assets in equilibrium. Therefore, to compensate for the fall in the tax base –labour and capital income–, tax rates must rise. We observe that the tax rate τ increases from 1.74 percent at $\theta = 0.45$ to 1.94 percent at $\theta = 0.50$, then to 2.22 percent at $\theta = 0.55$.

The second observation from Table 1 is that the voluntary unemployment rate increases with a rise in the replacement ratio. This monotonic relationship can be seen in the fifth row of Table 1. For the benchmark case, we find a 2.5 percent voluntary unemployment rate. With a 5 percent increase in the rate of replacement to $\theta = 0.50$, we find that the voluntary unemployment rate increases to 2.84 percent, that is, a 5 percent increase in the UI benefits leads to a further 0.34 percent of the voluntary unemployment rate. Finally, a 10 percent increase in the replacement ratio to $\theta = 0.55$ generates a 6.81 percent unemployment rate in the economy, which implies another 0.47 percent of the voluntary unemployment rate. The rate of increase in the voluntary unemployment rate is not linear with respect to the increase in θ. That is, higher rates of replacement imply higher voluntarily unemployed consumers, in an accelerating way.

An interesting observation regarding the effects of the UI system on the working decisions lies with the duration of the unemployment. We measure the duration of the unemployment by the percentages of unemployment spells. Since we have assumed that consumers are eligible only if they have worked in the previous period, they must not voluntarily opt out of job offers for two or more consecutive periods if they want to be eligible to enjoy higher level of the UI benefits. Consequently, they tend to reject job offers for a single period, enjoying more leisure time and a relatively acceptable level of the after-tax UI benefits. This generates a higher percentage of one-period unemployment spell with higher level of replacement ratio, as demonstrated by 85.03% with $\theta = 0.45$, 85.61% with $\theta = 0.50$ and 86.40% with $\theta = 0.55$. With a higher percentage of consumers unemployed for a single model period and a lower percentage of consumers unemployed for two or more model periods, we can conclude that a more generous UI system tilts unemployment to shorter duration. In other words, effects of a more generous UI system on labor market tend to be short-term instead of long-term.

Another steady state finding that we are going to discuss concerns endogenous asset cutoff points, which help us understanding the consumers' work/leisure decisions. Since we assume consumers enter job market at age 25 and leave at age 64, asset cutoff points are categorized into 160 different age groups given a quarterly model period and 40 years of admissible working time. If his asset holding a is larger than or equal to this asset cutoff point, that is, if $a \geq \Omega\bar{y}$, then he rejects the job offer. At time j, with $\bar{y}$ as the annualized output which is assumed to be relatively constant, Ω can be written as $\Omega(a, \varepsilon_j, \varepsilon_i, \theta, Age_j)$. We use ε_j to denote the job status η_{j-1} in the previous period and ε_i to denote the job offer in the current period, for $i, j = 1, 2, ..., 7$. If a consumer is unemployed in the previous period $j - 1$, his asset cutoff point function can be written as $\Omega(a, 0, \varepsilon_i, \theta, Age_j)$.

In equilibrium, no consumer rejects a job offer if he belongs to type $(a, 0, \varepsilon_i)$, regardless of his age. That is, if a consumer is unemployed in the previous period and he has a job offer with productivity ε_i in the current period, he will always take the job offer and join the group of employed people, regardless of his age.

The asset cutoff point function Ω is clearly non-decreasing in ε_i, where i is the index for the productivity level of current job offer with a larger i implying a higher productivity level. The higher the i is, the higher the asset cutoff point is. It means that consumers are likely to accept job offers with higher productivity thus higher wage compensations. On the other hand, Ω is non-increasing in ε_j, the productivity index for the previous period's job. The lower the index j thus the lower wage income due to lower productivity in the previous period, the higher the probability for consumers in the current period to accept the job offer because the UI benefits are relatively low with a low index number j.

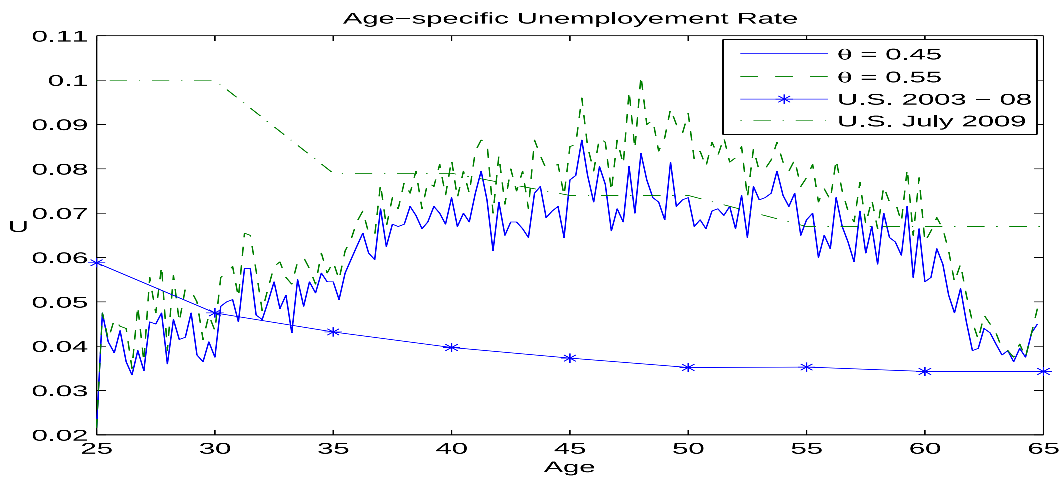
Within each age group, the higher the value θ is, the lower the asset cutoff points for each types of consumers. Essentially, Ω is non-increasing in the replacement ratio θ. If consumers are eligible to receive more UI benefits, they definitely become more likely to reject job offers than when they can only get a lower level of the UI benefits, all other factors fixed. For example, consumers of type $(a, \varepsilon_7, \varepsilon_6, Age_{64})$ have worked with the highest-paid job ε_7 in the previous period, and has a job offer in the current period with productivity ε_6. We can calculate the cutoff point for this special type of consumers in equilibrium. At $\theta = 0.45$, a consumer of this category will reject the job offer whenever his asset holding is higher than 4.7 times of his annual income; however, the same consumer will reject this job offer if his asset holding is higher than 2.8 times of his annual income under $\theta = 0.50$. When $\theta = 0.55$, this asset cutoff point function Ω is given by the lower bound of asset, $\underline{a}$. In this last scenario, returns from the UI benefits from previous period's labor income on job ε_7 is found to outweigh the labor income in the current period if the consumer chooses to take the job with productivity ε_6. In this way, we obtain a declining asset cutoff point function Ω with respect to θ.

In the steady state, a higher replacement ratio will lower the asset cutoff points for consumers, which in turn generates a higher level of the voluntary unemployment rate. This monotonic negative relationship between the UI replacement ratio and the asset cutoff points is the source of a larger voluntarily unemployed population observed in the country with higher θ.

In Figure 2, we plot for the age-specific unemployment rates for the economy with $\theta = 0.45$ with the dash line and $\theta = 0.55$ with the solid line. In the same Figure, we also plot the recent age-specific unemployment rates data (July - 2009),as well as the average age-specific unemployment rates for 2003-2008 in the U.S.

Overall, the solid line (representing the unemployment rates with $\theta = 0.55$), is always above the dash line (representing the unemployment rates with $\theta = 0.45$), implying that the voluntary unemployment rates are higher under a more generous UI system for *all* ages of consumers. Furthermore, the unemployment rates roughly have an inverted U-shape, with low unemployment rates among the young and old, and high unemployment rates among the 50 years old consumers. Young consumers enter the labour market with no assets, and therefore, accept whatever job offers available to them. The unemployment rate peaks roughly when the average asset holding peaks. Wealthier consumers choose not to work (and enjoy leisure) in the presence of low productivity shocks, as they can use their assets to finance current consumption. Interestingly, average wage income (i.e. average labor productivity) also peaks around early 50s. Why do consumers in their early 50s reject job offers with higher pay more often than those in 20s facing job offers with higher pay? The answer lies in the fact that the UI benefit is a function of the *previous* period's wage of the worker. Thus, while average wages rise with the age of the worker, so does UI benefit. This provides consumers in their 50s an additional incentive to reject wage offers more frequently. When consumers are older and closer to retirement, they become more likely to accept job offers in order to save for the retirement. Thus, while moral hazard due to UI benefits gives rise to moderate responses in terms of employment, its effects are on labor productivity and output are more substantial.

Figure 2: Age Specific Unemployment Rate



As stated earlier, a higher replacement rate implies a higher unemployment rate for all age groups. However, the *degree* of the moral hazard in response to higher replacement rates is age-dependent. As seen Figure 2), the most apparent difference between the dash and solid lines happens at ages between 48 to 53, indicating that moral hazard is most severe for those in early 50s. In fact, for the youngest consumers, the effects of the generosity of UI benefits are quantitatively negligible. These results suggest that a change in UI policy is unlikely to reduce high unemployment rates among the young households.

We have ruled out human capital accumulation in analyzing the effects of the UI benefits. If a consumer accumulates human capital on the job, he will be less likely to reject job offers since being unemployed will hurt his human capital development. In this sense, rejection rate will be lower for everybody in our model, especially for younger consumers. Since we have already found underestimation of the empirically high unemployment rates for young consumers, the assumption of human capital will further decrease the voluntary unemployment rates for the group of young consumers. Consequently, the age-specific unemployment rates that we have obtained can be deemed as the upper bounds.

At what level should the Canadian replacement ratio θ be in order to explain the average Canadian unemployment rate during the postwar period? In equilibrium, we find that *θ takes the value of 0.65.* The predicted unemployment rate at this level of UI benefit is given by 8.55 percent, which is roughly equal to the average unemployment rate in Canada between 1982 and 2004. That is, we need a replacement ratio at the level of two-thirds to have a large population that will be voluntarily unemployed. This higher replacement rate may reflect the ease at which consumers can “qualify” for UI benefits in Canada relative to those in the U.S.

4.2 Welfare and Distributional Effects

First, we report the welfare effects of UI benefits on the youngest consumers (with $j = 1$) in the economy. As these consumers start their economic life with no assets, welfare calculations for these consumers across different replacement rates are not affected by the aggregate asset holdings. Results indicate that the consumption value of generous UI payments in Canada is approximately 0.05% and 0.08% higher than the UI payments in the U.S. Welfare gains for a newcomer to the labour market from generous UI payments in Canada are largely offset by the tax increases to finance these UI programmes. We report the steady state percentage changes in welfare in Table 2.[8]

We also provide the welfare changes for consumers at the age 40 and 55.[9] This allows us to contrast potential differences in desirability of UI programmes for different age groups. In particular, we ask the following question: do wealthy consumers in their 50s value a generous UI programme as much as poorer young households?

Results indicate that a more generous UI system improves welfare for almost everybody

[8] Superscripts u and e correspond to those unemployed and employed consumers respectively. A positive value implies that consumers *prefer* the more generous UI system relative to the benchmark case.

[9] Unlike the youngest consumers, these consumers' average asset holdings are endogenous, and therefore, can bias the welfare calculations.

in the economy. Consumers will enjoy a higher level of insurance with a generous UI system even though tax rates also rise in equilibrium.

Table 2: Welfare Effects

Percentage $\triangle$ in Welfare	$\theta = 0.50$	$\theta = 0.55$
C^u	0.06%	0.24%
C^e	0.05%	0.08%
C	0.05%	0.08%

According to the model's predictions, unemployed consumers benefit the most from the generosity of UI benefits. The welfare gain is only 0.06% with a 5 percent increase of the replacement ratio from the benchmark case. However, if the replacement ratio is 10 percent higher than the benchmark case, the welfare gain for the unemployed consumers is around 0.24 percent, which is almost three times larger than that for employed consumers. As a whole, Canadian UI system is preferred by consumers due to the insurance gains from the higher replacement ratio.

We have assumed that the newcomer does not have any asset when he first enters the labor market so that $a = 0$. Based on the age-specific saving profile in Figure **??** to be discussed later, consumers of around 55 years old have the highest level of average asset holdings. Equilibrium consumption changes are outlined in Table 3. Relative to those just entering the labour market, older consumers' welfare respond more strongly to changes in the replacement rate. In general, social welfare improves upon more generous UI benefits. However, we also find that unemployed consumers are worse off with a higher level of UI benefits. When the replacement ratio increases from 0.45 to 0.55, 55 years old unemployed consumers prefer the textitlower replacement rate. In this sense, the welfare implications are not uniform for consumers of different ages. We conclude that since consumers differ in their asset holdings as well as age-dependent wage incomes, the UI system does not carry the same value to each consumer.

There are two opposing effects that affect the welfare of the unemployed consumers. First, when the replacement ratio θ increases from 0.45 to 0.55, the government pays more to eligible unemployed consumers. Consequently, welfare for these consumers is higher given a more generous UI scheme, as more insurance is provided. On the other hand, consumers *reduce* their asset holdings as they rely less on self-insurance. Thus, the average wealth drops for the unemployed consumers when the replacement ratio increases

Table 3: Welfare Effects for Consumers of age 40 and 55

	$\theta = 0.45$ (benchmark)	$\theta = 0.50$	$\theta = 0.55$
Percentage $\triangle$ in Welfare at age 40			
C^u	–	2.39%	0.19%
C^e	–	0.13%	0.97%
C	–	0.16%	0.59%
Percentage $\triangle$ in Welfare at age 55			
C^u	–	-3.11%	-6.22%
C^e	–	0.49%	0.90%
C	–	0.15%	0.24%
Statistic for Unemployed age 55			
Mean Asset	\$80,597	\$77,147	\$76,043
Median Asset	\$67,900	\$64,312	\$62,932

to 0.50 and further to 0.55, from 0.45. According to Table 3, the average asset savings for the unemployed consumers with $\theta = 0.45$ is \$80,597 , which is equivalent to 2 times of their average labor income in 2002 in the U.S. As the replacement ratio increases to 0.50, the average wealth decreases to \$77,147, and further declines to \$76,043 when the replacement ratio increases to 0.55. In steady state, asset-poor consumers have lower welfare all else equal. Our results indicate that the latter effect dominates the former, and therefore, we observe a lower welfare for the 55 year old consumers.

In order to eliminate the impact of endogenous wealth accumulation on welfare for the older consumers, we do the following calculation. We compute the welfare for a 55-years old, unemployed consumer in the economy with $\theta = 0.55$ with an asset level that is identical to the average asset level when $\theta = 0.45$. With this fixed asset level, an increase in the UI benefits *does* generate a higher level of welfare to 55-years old consumers.

Since welfare improves by comparing two steady state equilibria without taking into account of transition process, the estimates that we have arrived at are presumably the lower bounds of the welfare improvement. If we analyze the transition processes between these two steady state equilibria in the U.S. and Canada, welfare improvement will be higher since consumers would be able to increase consumption while reducing their asset holdings.

To illustrate the effect of a more generous UI system on wealth distribution within different economies, we calculate the Gini coefficients for wealth, income and consumption and outline them in Table 4. First, we observe that the Gini coefficients increase with respect to the replacement ratio, θ. That is, the distributions of wealth, income as well as consumption become more uneven with a higher replacement rate, θ. The changes in the distribution of income are attributed to the fact that a larger percentage of consumers voluntarily turns down job offers in Canada (i.e. when $\theta = 0.55$) than in the U.S. (i.e. when $\theta = 0.45$). In turn, these consumers have lower income, and have less to save. Thus, a larger fraction of population tends to wealth-poor as well. It is perhaps interesting to ask why consumption inequality worsens while welfare increases with a replacement rate. The answer to this question is given by the fact that consumers' welfare is positively affected by increased leisure of consumers during unemployment. This effect dominates the rise in consumption inequality, and thereby, improves welfare on net. The last entry at the bottom of the table is the conditional correlation between the wealth and the labor income for three different levels of UI benefits. With more voluntarily unemployed consumers, wealth and labor income becomes less correlated under more generous UI systems.

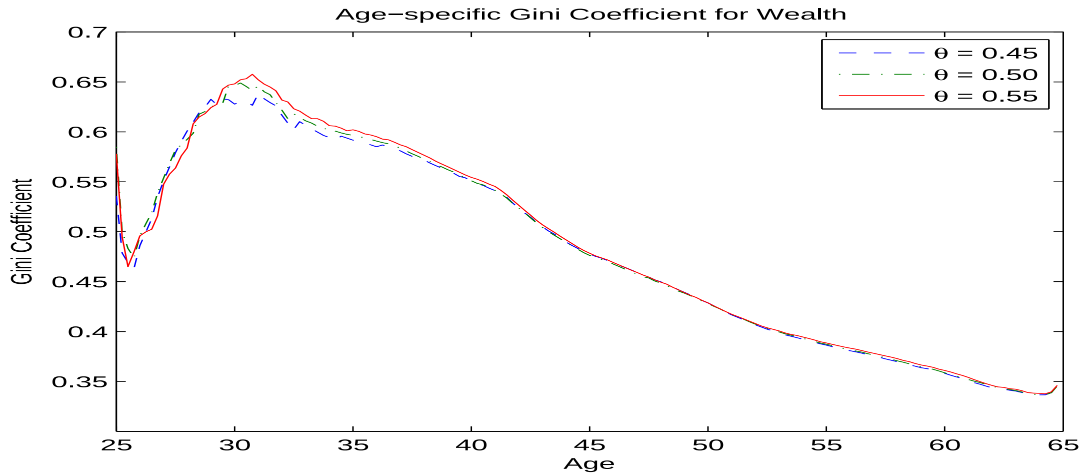
Table 4: Distributional Effects

	$\theta = 0.45$ (benchmark)	$\theta = 0.50$	$\theta = 0.55$
Gini coefficient of:			
wealth	0.4807	0.4829	0.4846
income	0.4200	0.4209	0.4223
consumption	0.3753	0.3762	0.3768
Correlation between			
(wealth, labor income)	0.7170	0.7167	0.7163

The Gini coefficients and the correlations between wealth and income obtained thus far are measured across consumers of all ages. It is interesting to analyze the age-profile of these statistics. Figure 3 plots the age-specific Gini coefficients for wealth. After a initial drop, the Gini coefficient increases rapidly with age, and reaches a maximum when the age reaches 32, roughly 7 years after consumers enter the labor market. Then the coefficient drops slowly, in a nearly monotonic way. That is, the most significant skewness for the distribution of wealth in the economy is for consumers aged between 30 and 45 years old.

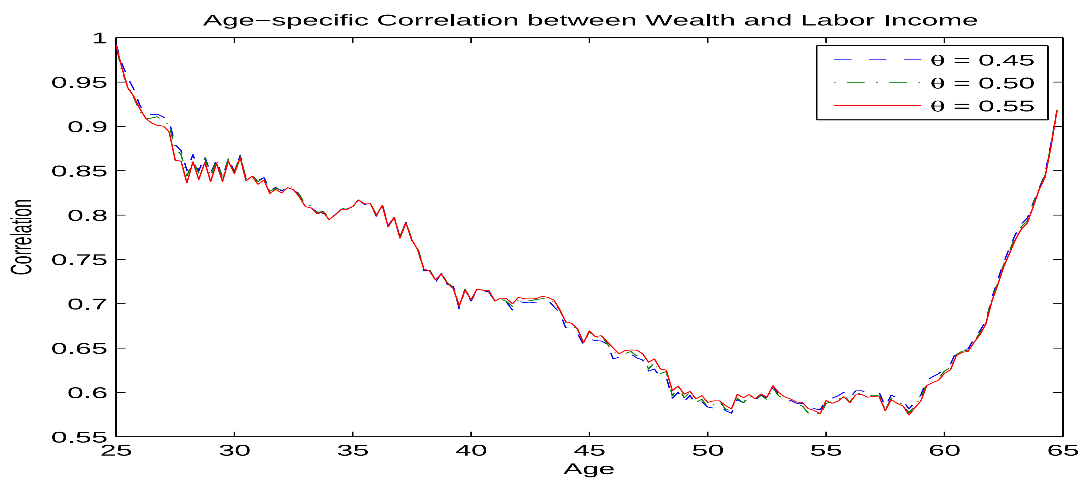
For the age-specific correlation between the wealth and the labor income, the plot mimics that of a U shape. The highest value is obtained for the newcomers to the labor market since these consumers have zero assets when they enter the labour market for the first time. As these once newcomers become older, they will accumulate some wealth,

Figure 3: Age Specific Gini Coefficient



and the correlation between the labor income and the wealth will drop gradually, until reaching a minimum when consumers are between 50 to 58 years old. After that, the age-

Figure 4: Age Specific Correlation



dependent wage income decreases monotonically, and the correlation picks up again. The plot is shown in Figure 4. The correlation overestimates the empirical findings made by Díaz-Gimenez et.al (2002). In their paper, the correlation between earnings and wealth is 0.47, and the correlation between the income and the wealth is found to be 0.60. We find that the correlation is around 0.72 (see Table 4). Moreover, from Figure 4, the minimum age-dependent correlation is approximately 0.55. The reason for this overestimation lies with the fact that our model does not have retired consumers in our economy whereas retirees exist in their model. Typically, retirees have large amounts of wealth but have nearly zero income in their study, reducing the wealth-income correlation on average.

4.3 Implications on Output and Saving

In this section, we report the effects of the UI system on aggregate savings, labor, consumption as well as output. Table 5 outlines the steady state percentage deviations from the benchmark statistics.

Table 5: Aggregate Outcomes

	$\theta = 0.45$ (benchmark)	$\theta = 0.50$	$\theta = 0.55$
Percentage Deviations in:			
K	–	-0.37%	-1.38%
N	–	-0.09%	-0.24%
Y	–	-0.33%	-0.58%
C	–	-0.19%	-0.64%
Volatility in Consumption			
σ_c	0.1458	0.1447	0.1431

First of all, given more UI benefits, consumers will save less under the same credit restriction that borrowing is not allowed. Consequently, we observe that the capital stock, K, declines by 0.37 percent if the replacement ratio increases to 0.50, and by 1.38 percent if the replacement ratio increases to 0.55 (see Table 5).

Since more consumers choose to turn down job offers, the average labor input N drops with regard to the increase in the UI benefits. We find a monotonic but accelerating drop in this value with every 5 percentage points increase in θ, similar to the observations on the voluntary unemployment rates. Given the drops in both aggregate capital and labor input, the aggregate output decreases with a more generous UI system.

We also observe that aggregate consumption, upon increases in the UI benefits, declines monotonically as well. As consumers turn down job offers more frequently, their lifetime (average) income drops, reducing their lifetime (average) consumption. Consumption volatility, measured by the variance of consumption (σ_c in Table 5), drops as θ increases. Thus, a relatively more generous UI system allows consumer to smooth their consumption fluctuation in a more effective way. Those who are separated from jobs can enjoy higher consumption levels with a higher replacement rate, θ, which reduces consumption volatility. Note that those who reject their job offer and have to reduce their consumption in that period relatively which increase consumption volatility. The former dominates in equilibrium, and therefore, we observe less consumption volatility at the aggregate.

5 Robustness Analysis

In this section, we carry out the robustness analysis along two dimensions. First, we fit the welfare income assistance ϕ to that in Canada under $\theta = 0.50$ and $\theta = 0.55$, then repeat the analysis of the impacts of the UI systems on consumers' optimal decisions. Second, we want to analyze whether a less stringent credit market can alter the effects of the UI benefits. In order to do this, we relax the condition that consumers must have nonnegative savings. Instead, they could borrow to finance their consumption if necessary. The limit of borrowing is assumed to be half of the steady state average wage, that is, $\underline{a} = -\frac{w}{2}$.

With ϕ calibrated to the Canadian income assistance, we find that a more generous UI system leads to more voluntary unemployment, with magnitudes similar to those obtained under the assumption that consumers are eligible for the U.S. income assistance. As consumers are better insured with a more generous UI system, they save less and their average per capita output is less than their counterparts in U.S.. However, due to the interaction of a more generous UI system and a less generous income assistance scheme in Canada, the magnitudes of the decline in savings as well as output are comparatively less significant, and welfare implications are mixed.

Under the assumption that consumers could borrow to finance their consumption, a more generous UI system induces more people to voluntarily turn down job offers. For example, a 5 percent and 10 percent increases in the replacement ratio can increase the voluntary unemployment rate by 0.30 percent and 0.74 percent respectively. The social welfare improves, and consumers will save less due to higher level of government insurance. Quantitatively, the relaxation of the credit market restriction does not affect the implications of the UI benefits in a significant way.

5.1 Welfare Assistance ϕ

In the benchmark case, we calibrate the income assistance or welfare assistance ϕ to the ratio of minimum wage and the income assistance in the U.S. This ratio is equal to 2.44. In this section, we calibrate ϕ to the ratio in Canada, which is equal to 3.05. Since the welfare assistance ϕ is inversely related to this ratio, a higher ratio implies a smaller value for ϕ, i.e., income assistance is comparatively more generous in the U.S. for consumers who are unemployed consecutively for more than two periods. Under this new ϕ, we want to measure the implications of the generosity of the UI systems. Using the same method as in benchmark case, the steady state equilibria are calculated for both $\theta = 0.50$ and $\theta = 0.55$ with income assistance calibrated to the Canadian economy. The equilibrium results are reported in Table 6.

The quantitative implications of the UI systems on the labor markets are very close in magnitudes to those with a higher income assistance ϕ. Higher levels of the UI benefits still encourage more consumers to voluntarily turn down job offers, and therefore, make a higher tax rate necessary. Consequently, the government rises the equilibrium tax rates from 1.74 percent at $\theta = 0.45$ to 1.94 percent at $\theta = 0.50$ and to 2.22 percent at $\theta = 0.55$. Given the stationary 3.5 percent of the involuntary unemployment rate, a 5 percent increase in θ generates another 0.35 percent voluntary unemployment rate compared with that when $\theta = 0.45$. Another 5 percent increase to $\theta = 0.55$ increases the voluntary unemployment rate by an additional 0.53 percent, with resulting overall unemployment rate at 6.88 percent. Comparatively, the equilibrium unemployment rates are slightly higher in magnitude than those under a higher welfare income assistance ϕ.

With increase in the generosity of the UI benefits, the duration of unemployment spell narrows. As could be seen from the percentages of the unemployment spells, percentage of consumers who are unemployed for a single period increases to 85.62 percent and 86.50 percent respectively when θ increases by 5 and 10 percent respectively from $\theta = 0.45$. This is due to the fact that consumers have to work in the previous period in order to qualify for the UI benefits in the current period. Thus they choose to be voluntarily unemployed for one period and enjoy a high level of the UI benefits, then accept whatever job offers in the period following.

Higher unemployment rates from a more generous UI system can be attributed to the fact that the asset cutoff points decrease with respect to the increase in θ. Using the same notion, Ω is non-increasing upon θ, making it possible for a larger group of consumers to

Table 6: Results with different ϕ

	$\theta = 0.45$(benchmark)	$\theta = 0.50$	$\theta = 0.55$
Tax rate $\tau\%$	1.74	1.94	2.22
Unemployment rate $\overline{U}\%$	6.00	6.35	6.88
Involuntary unemployment rate	3.50	3.50	3.50
Voluntary unemployment rate	2.50	2.85	3.38
Percent of unemployment spells			
1-period-unemployed	85.03	85.62	86.50
2-period-unemployed	10.02	9.63	9.04
3-period-unemployed or more	1.97	1.89	1.79
Percentage $\triangle$ in Welfare			
C^u	–	-5.27%	-5.11%
C^e	–	-0.02%	0.02%
C	–	-0.08%	-0.03%
Percentage $\triangle$ in Welfare at age 40			
C^u	–	2.76%	0.35%
C^e	–	0.04%	1.09%
C	–	0.11%	0.71%
Welfare change at age 55			
C^u	–	-3.10%	-6.88%
C^e	–	0.54%	1.05%
C	–	0.21%	0.35%
Gini coefficient of:			
wealth	0.4807	0.4813	0.4824
income	0.4200	0.4208	0.4224
consumption	0.3753	0.3759	0.3765
Conditional correlation between			
wealth, labor income	0.7170	0.7166	0.7162
Aggregate Outcomes			
K	–	-0.16%	-1.05%
N	–	-0.09%	-0.24%
Y	–	-0.12%	-0.54%
C	–	-0.20%	-0.46%
Consumption Volatility, σ_c	0.1458	0.1448	0.1433

possess asset which can surpass the asset cutoff point, and thus, reject the job offer. For example, the asset cutoff point for consumers of type $(a, \varepsilon_7, \varepsilon_6, Age_{64})$ is around 2.83 times of the average annual per capita output when $\theta = 0.50$. But this cutoff point drops to

$\underline{a}$ with $\theta = 0.55$, implying that the UI benefits together with the leisure time completely outweigh the return from working. Hence consumers of this type always reject job offers at $\theta = 0.55$. Moreover, the age-specific unemployment rate shows an inverted U shape, as in Figure 2. Still, this prediction is different from that observed empirically.

With respect to the welfare implications, the distributions of wealth, income, as well as consumption all skew to the right when θ increases. The conditional correlation between wealth and labor income decreases, since more income comes from the UI benefits instead of the labor income. The absence of retired consumers in our economy underlies this overestimation of correlation. Applying the same method to calculate the constant level of consumption, we find that there does not exist a monotonic improvement or deterioration of the welfare measured by consumption unit. Unemployed consumers seem to unanimously object to live in Canada compared with the U.S.. This is understandable since the income assistance ϕ is higher in Canada, which indicates that the U.S. is more generous to those who are consecutively unemployed for two or more periods, as could be seen from the actual data. For the employed consumers, their willingness to give up consumption seems to be symmetric around zero when the replacement ratios switch values between 0.50 and 0.55. Overall, welfare deteriorates for all consumers under this new income assistance system even with a more generous UI scheme. This mixed result might be due to the interaction of a more generous UI system in Canada but a more generous income assistance system in the U.S.. Consumers who can easily obtain a job prefer the Canadian system, but those who can not easily locate a job prefers to stay in the U.S., where they can receive a higher level of income assistance even when they are unemployed for all their lives. The welfare measured in consumption unit for consumers of age 40 and 55 has similar qualitative implications as those in Table 3. It is evident that the welfare implications are not totally independent of the asset holdings of consumers. When the consumers do not have any asset, they would appreciate more UI benefits. This condition will change when they possess some assets as they become older so that they do not appreciate a more generous UI system as much as before. The unemployed consumers under a higher rate of replacement are poor in wealth, and this effect will dominate the welfare-improving effect of a higher level of the UI benefits. If we fix the asset level, we will see that a more generous UI scheme under the life-cycle model is welfare improving. Measuring in the consumption unit, all statistics are positive, in a similar magnitude to those in Table 3.

The qualitative effects of a more generous UI system on output as well as savings are similar to those obtained with a higher income assistance. Consumers will save less, supply less time and thus a lower output will be produced. However, since they are getting lower amount of income assistance when they are unemployed for more than two consecutive periods, decreases in their savings as well as output are smaller by comparison. This is shown by the entries in Table 5 and those at the bottom of Table 6. Still, a higher θ can serve as a better buffer for those unemployed for one period, as implied by a declining σ_c with increase in θ. The age-dependent profiles of savings as well as consumptions share similar shape as those with a higher income assistance.

In summary, the implications of a more generous UI system together with a lower income assistance scheme are close in quantitative magnitudes to those with both a higher level of the UI benefits and a higher level of income assistance. There are a few more voluntarily unemployed consumers, who will save less, consume less, and enjoy more leisure time. Consequently, equilibrium results are not found to be sensitive to the specifications of income assistance ϕ.

5.2 Credit Limit $\underline{a}$

In this section, we proceed our study by assuming that consumers can borrow up to half of their wage income. The weight for leisure λ is calibrated to be 0.10 in the steady state, and the values for other parameters are approximately equal to those under the assumption that consumers cannot borrow. We outline the equilibrium results in Table 7.

According to Table 7, under the assumption that consumers can obtain more credit to finance consumption, a more generous UI system increases the voluntary unemployment rate in a magnitude similar to those when consumers are not allowed to borrow. With an increase in θ to 0.50, we observe a 0.30 percent increase in the voluntary unemployment rate, and a further increase in θ to 0.55 generates an additional 0.44 percent voluntary unemployment rate. Notice that the equilibrium response of unemployment to increases in θ is slightly muted relative to that in the benchmark calculations. The ability to borrow gives consumers additional flexibility when considering to reject low-paying job offers. This flexibility also renders the effects of replacements on unemployment less noticeable.

With respect to the age-dependent unemployment profile, the unemployment rates for young consumers are still too low compared to the actual unemployment rates. The duration of unemployment, denoted by the percentage of unemployment spells, declines with respect to the increase in the generosity of the UI system. This implies a short-term effect of the UI benefits, where consumers take advantage of a high level of the UI

Table 7: Results with $\underline{a} = -w/2$

	$\theta = 0.45$(benchmark)	$\theta = 0.50$	$\theta = 0.55$
Tax rate τ%	1.74	1.94	2.21
Unemployment rate $\overline{U}$%	6.00	6.30	6.74
Involuntary unemployment rate	3.50	3.50	3.50
Voluntary unemployment rate	2.50	2.80	3.24
Percent of unemployment spells			
1-period-unemployed	85.01	85.56	86.31
2-period-unemployed	10.04	9.68	9.16
3-period-unemployed or more	1.98	1.89	1.82
Percentage $\triangle$ in Welfare			
C^u	–	0.06%	0.24%
C^e	–	0.06%	0.10%
C	–	0.06%	0.10%
Percentage $\triangle$ in Welfare at age 40			
C^u	–	1.98%	-0.02%
C^e	–	0.10%	1.07%
C	–	0.13%	0.68%
Welfare change at age 55			
C^u	–	-3.16%	-5.47%
C^e	–	0.51%	0.80%
C	–	0.18%	0.26%
Gini coefficient of:			
wealth	0.4814	0.4830	0.4849
income	0.4199	0.4207	0.4221
consumption	0.3753	0.3760	0.3767
Conditional correlation between			
wealth, labor income	0.7170	0.7161	0.7158
Aggregate Outcomes			
K	–	-0.16%	-1.22%
N	–	-0.09%	-0.21%
Y	–	-0.10%	-0.58%
C	–	-0.19%	-0.46%
Consumption Volatility, σ_c	0.1459	0.1448	0.1430

benefits for a single period and take whatever job offers in the period that follows. This phenomenon is due to the qualification condition that consumers are eligible for the UI benefits in the current period only if they have worked in the previous period.

The welfare effects of UI system are quantitatively close in magnitudes to those when the consumers have no access to the credit markets. The relaxation of the borrowing limit has quantitatively no impact on the welfare calculations we obtained for the benchmark economy. This result indicates that the self-insurance provided by risk-free bonds does not fully offset the insurance benefits of the IU system. Due to the consumption smoothing effects for the consumers who are involuntary unemployed, a more generous UI system allow consumers of all ages to smooth their consumption more effectively.

The distributions of income, as well as wealth and consumption, are skewed to the right with a higher level of the UI benefits. Furthermore, the labor income and the wealth become less correlated given an increase in the UI benefits. This is because wealthier consumers choose to reject the job offers and receive the UI benefits, reducing the correlation between wealth and income. The age-dependent profile for this correlation drops to the lowest level for consumers between 50 and 58 years old, during which the age-dependent unemployment rates peak.

Together with the ease of the credit market, a more generous UI scheme induces consumers to save less, work less, and thus produce less. They will enjoy more leisure time and consume a smaller amount of good. Quantitatively, these responses are close to those when consumers cannot borrow.

Overall, the implications of the UI system on working decisions, saving as well as consumption, when consumers have access to credit markets in both countries, are very close in magnitudes to those when they are not allowed to borrow. There will be a small increase in the voluntary unemployment rate, a decrease in saving and output, together with an improvement in social welfare.

6 Conclusion

In this study, we assess the effects of the UI benefits on the unemployment rates in Canada and the U.S. in the last three decades. We find that, while the relatively higher UI replacement rates in Canada cannot fully explain the difference in average unemployment rates in two countries, they induce especially older (wealthier) consumers to reject job offers in quantitatively significant way. For example, an increase in the replacement ratio from 45% to 50% produces a 0.30 percent increase in the unemployment rate, and generates another 0.50 percent unemployment rate when the replacement ratio is increased to 60%.

We also find that a higher level of UI benefits provides more insurance to the unemployed, and therefore, consumers reduce their asset holdings as their demand for self-insurance decreases. Their lower labor input together with a smaller capital stock implies a lower output. However, this decrease is not that large in magnitude. For instance, a 10 percent increase in the replacement ratio will cause a reduction of per capita output by approximately 0.60 percent.

Consumers of different ages save in different amounts. In equilibrium, we find that average savings are low when consumers are young, peaking in their middle ages and falling when they are close to retirement. This observation can be attributed to the fact that wage income peaks around when consumers reach the age of 50. Regarding the unemployment rates, young consumers have lower levels of unemployment rate, because they are both asset-poor and income-poor. For older people who are close to retirement, the motive for saving for the retirement is strong, and therefore, they will not turn down job offers frequently. The wealth and income richest consumers, who typically are in their early 50s, reject job offers more frequently than any other group. In addition, their job-rejection rate increases more substantially than that of any other age group when the replacement rate is increased.

There is one point where our prediction is different from that of the empirical observations. Our prediction in age-dependent unemployment rates underestimates the high unemployment rates for younger consumers. We attribute this discrepancy to the uniquely assumed involuntary unemployment rate in the model. Thus, we conclude that generous UI schemes are unlikely candidates to explain the observed high unemployment rates among the youth.

References

Andolfatto, D. & Gomme, P. (1996) "Unemployment insurance and labor-market activity in Canada." *Carnegie-Rochester Conference Series on Public Policy* 44, 47-82.

Athreya, K. (2008) "Default, insurance, and debet over life-cycle." *Journal of Monetary Economics* 55, 752-774.

Clark, K. & Summers, L. (1982) "The Dynamics of Youth Unemployment." NBER Working Papers 0274.

Costain, J. S. & Reiter, M. (2003) “Business cycles, unemployment Insurance, and the calibration of matching models.” CESifo Working Paper No. 1008.

Deaton, A., Gourinchas, P.-O. & Paxson, C. (2000) “Social security and inequality over the life cycle.” NBER Working Paper No. 7570.

Green, D. A. & Riddell W. Craig (1993) “The economics effects of unemployment insurance in Canada: an empirical analysis of UI disentitlement.” *Journal of Labor Economics* 11, S96-S147.

Gruber, J. (1997) “The consumption smoothing benefits of unemployment insurance.” *American Economic Review* 87, 192-205.

Hansen, G. (1993) “The cyclical and secular behavior of the labor input: comparing efficiency units and hours worked.” *Journal of Applied Econometrics* 8, 71-80.

Hansen, G. & İmrohoroğlu, A. (1992) “The role of unemployme insurance in an economy with liquidity constraints and moral hazard.” *Journal of Political Economy* 100, 118-142.

Huggett, M. (1996) “Wealth distribution in life-cycle economics.” *Journal of Monetary Economics* 38, 469494.

Joseph, G. & Weitzenblum, T. (2003) “Optimal unemployment insurance: transitional dynamics vs. steady state.” *Review of Economic Dynamics* 6, 869-884.

Obiols-Homs, F. (2003) “Incomplete unemployment insurance and aggregate fluctuations.” *Review of Economic Dynamics* 6, 602-636.

Díaz-Giménez, J., Quadrini, V., Ríos-Rull, J., and Rodríguez, S., (2002) “Updated facts on the U.S. distributions of earnings, income, and wealth.” Federal Reserve Bank of Minneapolis *Quarterly Review* 26, 2-35.

Shimer, R. (1998) “Why is the U.S. unemployment rate so much lower?” NBER Macroeconomics Annual 1998, volume 13.

Storesletten, K., Telmer, C. & Yaron A. (2004) “Consumption and risk sharing over the life cycle.” *Journal of Monetary Economics* 51, 609-633.

Tauchen, G. (1986) “Finite state Markov-chain approximation to univariate and vector autoregressions.” *Economics Letters* 20, 177-181.

Valletta, R. & Hodges, J. (2005) "Age and Education Effects on the Unemployment Rate" Federal Reserve Bank of San Francisco Economic Letter Number 2005-15.

Wang, C., and Williamson, S.D. (2002) "Moral Hazard, Optimal Unemployment Insurance, and Experience Rating." *Journal of Monetary Economics* 49, 1337-1371.

Appendix – Computation method

In total, we have six parameters to calibrate, i.e., the weight to leisure in utility valuation λ, the stationary involuntary unemployment rate $p(\tilde{u})$, the parameter to the wealth felicity function γ, and those governing idiosyncratic productivity shocks $(\rho, \sigma_m, \sigma_o)$. Weight λ is estimated to target the 6 percent unemployment rate, and $p(\tilde{u})$ is estimated to generate the distribution of the unemployment spells, both for the postwar U.S. economy. All others are calibrated to fit various age-specific criteria.

Our computation procedure can be divided into following steps:

1. At $\theta = 0.45$, assume values for $(\rho, \sigma_m, \sigma_o)$.

2. Assume positive number for γ, say, $\gamma = 1$.

3. Assume $p(\tilde{u}) = 0.01$, that is, there is a one percent involuntary unemployment rate in the economy.

4. Assume a positive weight for leisure, say $\lambda = 1.0$.

5. Guess a government tax rate τ, say $\tau = 0.05$.

6. With the tax rate τ, guess the return on capital r. Based on the first order condition of firm's profit maximization problem, calculate the average wage w and the implied demand-side capital-labor ratio.

7. Given the wage rate w, scale the age-dependent wage income $\{\mu_t\}_{t=1}^{J}$ so that the average of this wage income process is equal to the wage rate w. With this newly obtained age-dependent wage income process, construct the wages received by consumers if they choose to work under various labor productivity.

8. Solve backwardly from period J to period 1 for consumers' utility maximization problem. Keep the optimal decision rules for consumption, savings as well as working decisions for use in simulations.

9. Assume a certain number of consumers for each age group. For example, we assume that there are $M = 2000$ consumers within each group from age 25 to 64. Simulate a matrix of uniformly distributed random numbers of dimension $M \times J$, with each row correspond to a different consumer.

Together with the transitional probability matrix Γ, we derive the wages facing each consumer of each age. Simulate the economy with M consumers and obtain their decision rules on consumption, savings as well as working choices.

10. Aggregate over consumers of all ages to obtain the average capital K and the efficient labor input N. If the capital-labor ratio from this supply-side is equal to that ratio from the demand-side, the interest rate r is deemed as the equilibrium rate that clears the market and we can go to the next step. Otherwise, update r based on whether there is an excessive demand or supply and then repeat the steps from optimization to simulations.

11. With the clearance of markets at an equilibrium interest rate r, we check the balance of the government budget. If the total taxes T is equal to the total benefits B, then go to the next step. Otherwise, update τ and repeat all steps between the optimization and the simulations with the new government tax by method of bisection.

12. With the clearance of both aggregate constraint and government budget, check if the implied unemployment rate is equal to the postwar U.S. unemployment rate. If they are equivalent, go to the next step. Otherwise, update λ and repeat the above steps for clearance of both the markets and the government.

13. With the implied unemployment rate equal to the postwar U.S. unemployment rate and the clearance of markets as well as the balance of government budget, check whether the distribution of the unemployment spells from the simulated economy equals that observed in the postwar U.S. economy. If they are equivalent or close enough to each other, go to the next step. Otherwise, update $p(\tilde{u})$ and redo the above steps from assuming leisure weight λ to clearance of all markets and government's balance.

14. At this step, we need to check whether the ratio of the retirement savings to the labor income obtained from the model is equal to that in the data. If so, go to the next step; otherwise, update γ by bisection method and repeat all the above steps between assigning value to γ to the match of distribution of unemployment spells.

15. At this final step, if the volatilities of log income generated in the model resemble those in the data, the steady state is obtained and we could report equilibrium statistic. Otherwise, we need to update parameters for the productivity shock and repeat all steps above.

Since $p(\tilde{u})$ can not exceed 6 percent and can not be negative, we start from 1 percent involuntary unemployment rate as specified in the step of computations. In order to have a rough idea of how the unemployment spells behave with respect to this stationary involuntary unemployment rate, we select several grid points from 1 percent to 5.5 percent with grid size of 0.50 percent. The steady states are solved by using these involuntary unemployment rates. We find that at a certain involuntary unemployment rate level the distribution of the unemployment spells cease to change and remain in a relatively stable magnitude. Furthermore, we find that an involuntary rate of unemployment higher than four percent can not be used to calibrate to the postwar U.S. unemployment rate of six percent, within our framework.

As we are dealing with a finite life cycle optimization problem, value function iteration is not necessary. Given the final condition at time J, we face a deterministic problem and could solve it directly. Then using the value function for this last working period, we could solve for the value function one period earlier. We repeat this process until the first period when consumers enter the labor market.

The market clearing criteria are set so that the equilibrium is supposed to meet whenever change in the interest rate is less than 10^{-6} unit of zero or change in the capital-labor ratio is less than 10^{-5} unit of zero. For the government's budget, we deem that the equivalence is met when either the change in the tax rate or the change in excess demand. Difference in the implied unemployment rate and the postwar U.S. unemployment rate is deemed as negligible if it is less than 10^{-4} in absolute terms.